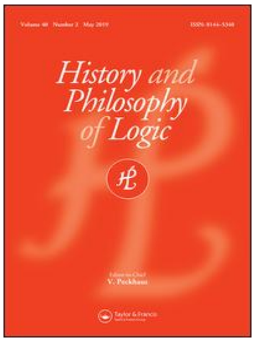

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Duty and Sacrifice: A Logical Analysis of the Mīmāṃsā Theory of Vedic Injunctions

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The Mīmāṃsā school of Indian philosophy has for its main purpose the interpretation of injunctions that are found in a set of sacred texts, the Vedas. In their works, Mīmāṃsā authors provide some of the most detailed and systematic examinations available anywhere of statements with a deontic force; however, their considerations have generally not been registered outside of Indological scholarship. In the present article we analyze the Mīmāṃsā theory of Vedic injunctions from a logical and philosophical point of view. The theory at issue can be regarded as a system of reasoning based on certain fundamental principles, such as the distinction between strong and weak duties, and on a taxonomy of ritual actions. We start by reconstructing the conceptual framework of the theory and then move to a formalization of its core aspects. Our contribution represents a new perspective to study Mīmāṃsā and outlines its relevance, in general, for deontic reasoning.

1. Introduction

1.1. *Presentation of the topic*

The Mīmāṃsā school of Indian philosophy provides an analysis of injunctions (*vidhi* in Sanskrit) that are found in a set of sacred texts called the Vedas. We use ‘injunction’ in the sense of a statement which enjoins someone to do something, thus expressing a *duty*. The injunctions with which Mīmāṃsā is concerned are generally injunctions to perform sacrifices (*karma*, or ‘action’, being used as the technical term for these ritual actions). When we speak of the ‘analysis’ of injunctions, we mean the elaboration of a set of general principles according to which a given injunction might be understood and hence put into action.[1]

The foundations of Mīmāṃsā as a philosophical school were laid perhaps in the second century before the common era. Mīmāṃsā provides some of the most detailed and systematic analyses available anywhere of statements with a deontic force. These discussions, however, have generally not been registered outside of Indological scholarship. One way to evaluate their philosophical significance is through a rigorous analysis of the deontic concepts that they employ. Such an analysis can provide important insights about their precise characterization, including the similarities to, and differences from, the way in which concepts such as ‘obligation’ and ‘permission’ have been defined in other philosophical systems. As a part of our reconstruction of the arguments employed by Mīmāṃsakas (philosophers operating within the Mīmāṃsā school), we introduce a formal system of reasoning focused on fundamental notions involved in these arguments, such as ‘sacrifice’,

[1] For a short introduction to Mīmāṃsā in general, see *Freschi* (*2017*). For injunctions and ritual duties as conceived in Mīmāṃsā, see sections 3–6 of *Freschi 2012* and *Ollett 2013*.

'primary action', 'subsidiary action' and 'eligibility'. This formal perspective can, in turn, offer new interpretive and philosophical insights into the texts of Mīmāṃsā in which the arguments at issue were articulated.

The formalization of deontic arguments in Mīmāṃsā should be seen as part of an ongoing attempt to use formal approaches to represent arguments and principles of reasoning in Indian philosophical thought. This enterprise has been going on since the beginnings of the study of the history of Indian logic in Europe (see, for instance, *Stcherbatsky 1930–1932*, *Ingalls 1951*, *Bocheński 1956* and *Staal 1967*) and the trend has continued up to recent years, especially in regard to Buddhist philosophy of the Madhyamaka school (see, for instance, *Garfield and Priest 2003* and *Guhe 2017*). Another area of application of symbolic tools has been the epistemology and logic of the Dignāga-Dharmakīrti school (see for instance *Patil 2009* and *Tillemans 2013*) and of the Nyāya and Nāvya Nyāya school (see for instance *Matilal 1985* and *Ganeri 2001*). However, no work has systematically examined the analysis of logical principles and, especially, principles of deontic reasoning in the Mīmāṃsā school.[2] In fact, the natural place to look for the study of deontic concepts in India is Mīmāṃsā, which is however far less studied than other philosophical traditions. Moreover, the few scholars working on Mīmāṃsā by using logical formalizations focused on epistemology and other topics that did not involve deontic concepts (see, for instance, *Yoshimizu 2007*).

Given the enormous breadth of Mīmāṃsā, a tradition which spans over two thousand years, and includes many thinkers who disagreed with each other, the research agenda of studying the principles of deontic reasoning used by Mīmāṃsākas must proceed on a problem-by-problem basis and must begin with a focus on particular texts. This paper, while addressing one specific, though central, aspect of the Mīmāṃsā theory, will also lay some of the groundwork for further research, and should be taken as a starting point in the logico-philosophical reconstruction of this theory. Furthermore, the present work will enable scholars to compare the contribution of Mīmāṃsā to the study of logic and reasoning with the contribution of other important schools of Indian philosophy, such as the Nyāya school and the Dignāga-Dharmakīrti school.

Finally, our analysis will deal with issues that are relevant to deontic reasoning in general, such as the difference between *strong* and *weak* duties.

1.2. Working plan for the present article

The main part of this article will focus on what we call *Common Mīmāṃsā*. The label refers to the early history of the philosophical system of Mīmāṃsā, as depicted in its root texts, Jaimini's *Mīmāṃsāsūtra* 'Maxims on Vedic exegesis' or 'Exegetic Aphorisms' (possibly 2nd c. BCE) and in Śabara's *Bhāṣya* 'Commentary' (possibly 3rd c. CE).[3] The specific problem that we address pertains to the classification of sacrifices into *fixed*, *occasional*, and *elective*, that is accepted in Common Mīmāṃsā. On the one hand, the injunctions to perform sacrifices of these three types have an identical linguistic form[4] and they are all regarded as originating a kind of duty. On the other hand, we will see that fixed

[2] For a recent and more generic discussion of deontic and related concepts in ancient India, see *Majumdar 2017*.

[3] See *Freschi and Pontillo 2013* for these dates. The title *Mīmāṃsāsūtra* has been differently translated. In general, it states that the text is about Mīmāṃsā, i.e. about an investigation on Vedic texts, and that it is composed in short synthetic and terse sentences, called *sūtra*s. These are often hardly understandable without an extended commentary, and *bhāṣya* means indeed 'extended commentary'. For pragmatic reasons, in the following we will refer to these texts with an English translation, i.e. as *Exegetic Aphorisms* and *Commentary on the Exegetic Aphorisms* respectively. See *Subbāśāstrī 1929–1934* for the full text of the *Exegetic Aphorisms* and of the *Commentary on Exegetic Aphorisms*.

[4] It may be appropriate to note here that Mīmāṃsā authors were well aware of the fact that the linguistic form of a statement is not a reliable predictor of its semantic value. Statements in the indicative can, in the appropriate context (e.g. a book of recipes)

and occasional sacrifices are compulsory in a way that elective sacrifices are not, and furthermore, the subsidiary actions that form part of an elective sacrifice, and those that form part of a fixed or occasional sacrifice, are also not compulsory in the same way. The taxonomy of ritual actions therefore poses a significant philosophical challenge: how exactly should the various notions of 'duty' at play in this taxonomy be characterized, and what accounts for their differences?

As we said, part of our reconstruction consists in employing a formal language. The rendering of complex arguments from the history of philosophy—and in particular from Sanskrit texts composed more than fifteen centuries ago—into a formal language is a difficult enterprise. There is a risk that what is rendered in the formalism will bear little relation to the textual argumentation, either because it is too abstract, or because important distinctions are omitted. To minimize this risk, we will proceed in two stages. *First* (Section 2), we will offer a descriptive analysis of the phenomenon, as far as we understand it from a set of texts that we consider to represent Common Mīmāṃsā. This will involve setting out the three main categories of sacrificial actions, and introducing some of the general principles that are invoked in differentiating them and eliciting their deontic consequences. *Second* (Section 3), we introduce a formal language based on this taxonomy and build a logical system in which it is possible to represent generic patterns of reasoning related to the Mīmāṃsā theory. We provide a semantic characterization of our system and illustrate some possible variations of it, such as extensions including further logical principles or different logical operators. Finally, we illustrate how the formal systems introduced here are related to a formal system that was previously used to model a debate within Mīmāṃsā (Appendix).

In addition to this, in Section 4, we discuss a thesis formulated by an eighth-century Mīmāṃsā author, Maṇḍana Miśra, who attempted to reinterpret the theory of sacrifices found in Common Mīmāṃsā. Maṇḍana thought that all Vedic injunctions can be redescribed in terms of a notion of instrumentality towards a given end, without losing the distinctions that arise from the traditional taxonomy. Maṇḍana's intervention is 'reductionist', both in the sense that it brings the types of duties pertaining to various kinds of sacrificial action under a uniform description, and also in the sense that it reduces deontic concepts to non-deontic concepts. We discuss how these simplifications affect the formalization we provided, and what philosophical consequences they have.

1.3. *Mīmāṃsā's particularities*

In this paper we will discuss the deontic concepts employed by authors in the Mīmāṃsā tradition. Here we must draw attention to three features of this tradition, so that the reader may have a clearer idea of what these concepts are, what it means for us to attribute them to Common Mīmāṃsā, and the role that individual authors played in defining and redefining this system. These features distinguish Mīmāṃsā from other traditions of reflection on deontic concepts, and hence should condition our expectations of Mīmāṃsā in comparison to other such traditions, such as Talmudic or Quranic interpretation, Euro-American traditions of moral philosophy, or Confucian ethics.

Deontic concepts are very often invoked in connection with an idea of what is 'good' or 'right', that is, in connection with ethical concepts. This connection is so weak in Mīmāṃsā that we may think of its deontology and ethics being completely independent from each other: by and large, Mīmāṃsā is concerned with analyzing rituals, the performance of

be in fact injunctions and injunctions can be equally conveyed by the optative, imperative, subjunctive modes, as well as by gerundive participles.

which is enjoined in the Vedas; it is taken for granted that the performance of these rituals is conducive to some notion of ‘the good’, which is, however, not determined with reference to any ethical viewpoint. Moreover, the very fact that an action is enjoined in the Vedas is taken to mean that it cannot be derived from any understanding of ‘the good’ that is independent of the Vedas. One example is the Vedic injunction ‘do not tell a lie’: in Common Mīmāṃsā, this is not a general ethical recommendation, but a specific requirement of someone who is performing the full- and new-moon sacrifices, having exactly the same deontic and ethical status as injunctions, such as ‘pour the ghee into the fire’, which transparently apply only to ritual actions.[5] The duties that we will be examining in this paper, therefore, pertain specifically to the sphere of ritual.

The stated goal of Mīmāṃsā is to interpret the statements of the Vedas, and thus to provide specific guidance for performing the rituals they enjoin. But there was widespread agreement, from a relatively early period, about what the Vedas tell us to do. By contrast, Mīmāṃsā authors often disagreed about the principles by which we are led to these interpretations. Thus, if we only look at the conclusions—that a particular person, in a particular circumstance, has a duty to perform a certain ritual action—it will appear as if Mīmāṃsā authors all agree with each other. But if we look at the reasoning leading up to those conclusions, we see significant differences, since some authors, like Prabhākara (6th–7th c. CE), interpret the Vedas as a set or rules to be followed, while other authors, like Maṇḍana, interpret the Vedas as a set of recipes to reach desired goals.

Finally, Mīmāṃsā, like other traditional knowledge systems in India, operates according to the assumption that all of the doctrines of the system are contained *in nuce* in the foundational work of the system, namely, Jaimini’s *Exegetic Aphorisms*, and that every subsequent author has merely explained, clarified, or summarized these doctrines. This is hardly more than a conceit, and in fact many authors have introduced radical changes into the system, but very often they have done so by claiming that their interventions are merely elaborations of ideas that can be found in earlier works. Accordingly, determining the specific contributions of each author is sometimes difficult, and even authors who have radically revised the system, like Maṇḍana, go out of their way to remain faithful to the distinctions and classifications introduced by earlier authors.

2. Vedic duties in Common Mīmāṃsā

Mīmāṃsā is principally concerned with *duties*: how they are to be understood and followed, on a practical level, and how they are to be theoretically grounded. But the set of real-world duties is much larger than the set of duties that Mīmāṃsā addresses. Mīmāṃsā is concerned specifically with those duties that are presented in the Vedas, and even more specifically, with those duties presented in the Vedas *to perform ritual actions*. Hence Mīmāṃsā, literally ‘analysis’, is often called *karmamīmāṃsā*, ‘the analysis of ritual actions’. We are principally interested in the types of duties that Mīmāṃsā has to contend with, but in order to discuss those types of duties, we must first discuss the types of ritual actions which are involved in Vedic duties.

Conceptually, Mīmāṃsā authors distinguish between a main *sacrifice* (for instance, the full- and new-moon sacrifices, the *darśapūrṇamāsa*) and each *rite* constituting it (for instance, an offering to the god Agni, one to the gods Agni and Soma jointly, one to the god Indra within the full- and new-moon sacrifices). They further distinguish between a rite (e.g. the offering of a rice-cake to Agni), and the actions constituting it (e.g. grinding the rice and sifting the flour), which can, again, entail distinct activities (such as putting

[5] This is discussed under the topic of ‘the agent’ in Śabara’s *Commentary* on 3.4.12–13. In South Asia, it has generally been religious and literary traditions, rather than philosophical systems, that have dealt with ethical questions; see *Matilal 2002*.

the rice-grains in the mill and pressing one stone against the other).[6] For the sake of the present article we do not have to analyze all levels of this hierarchy of actions; it is enough to distinguish between the (main) sacrifices and all the actions constituting them (from rites to specific activities). This distinction is at the basis of the differentiation between elective, occasional and fixed sacrifices (see Sections 2.1.1, 2.1.2 and 2.1.4). We will say that a sacrifice is a *primary* ritual action and that all its components are *subsidiary* ritual actions.

2.1. Description of the phenomena

2.1.1. *Types of sacrifices and ritual actions* On a practical level, Mīmāṃsā operates with a threefold distinction among types of sacrifices:

1. *Fixed sacrifices* (*nityakarman*): Those which one is obligated to perform recurrently throughout one's life.
   - 'As long as one lives, one must perform the *agnihotra* sacrifice.' (This is explicitly said to be obligatory for the duration of one's life.)
   - 'One should worship at dawn, noon, and twilight.' (The thrice-daily worship, *sandhyāvandana*, is also a lifelong duty.)
2. *Occasional sacrifices* (*naimittikakarman*): Those which one is obligated to perform when a given condition (the *nimitta* 'occasion' of the sacrifice) is met. Occasions are specific *events* happening in one's life and triggering the performance of a given sacrifice and can vary from the birth of a son to the fact of having broken a vessel during a given sacrifice. The Sanskrit word *nimitta* 'occasion' can also be translated as 'cause' and it evokes the one-to-one relation linking an occasion and the thing it occasions (called *naimittika*), in this case, the occasion and the sacrifice.
   - 'On the birth of a son, perform the *jāteṣṭi* sacrifice.'
   - 'On the occasion of a solar eclipse, perform the *grahaṇaśrāddha*.'

   Note that occasional sacrifices and fixed sacrifices are very similar, and often there is genuine disagreement about whether a particular sacrifice fits into one or the other category. The essential difference seems to be the *fixity* or *regularity* of the occasion upon which one is obligated to perform the sacrifice. Typically, *fixed* sacrifices have to be performed every day, while *occasional* sacrifices have to be performed only when something 'comes up'. However, sacrifices which take place at a specified time in the calendar year (such as the *darśapūrṇamāsa* sacrifices or the *jyotiṣṭoma* sacrifice) are predictable (unlike the prototypical occasional sacrifices) despite not recurring every day (unlike the prototypical fixed sacrifices).
3. *Elective sacrifices* (*kāmyakarman*): Those which one should perform if one has the desire for the result mentioned in the statements enjoining them.
   - 'One who desires cattle should sacrifice with the *citrā*.'
   - 'One who desires to kill his enemy should enchant sacrificing with the *śyena*.'

We emphasize that the threefold classification occurs on a *practical* level because its logical structure and implications were never systematically worked out in early Mīmāṃsā. In other words, it is assumed by the authors, as if everyone were familiar with it from other sources, rather than presented and analyzed explicitly. This classification does, however, have important consequences, for example, in determining how stringently the injunctions

[6] See, for instance, *Exegetic Aphorisms* 5.2.6 for a discussion of how each action entails several activities.

for performing the sacrifice are to be followed, as well as for understanding the desirable and undesirable results that attach to the performance or non-performance of the sacrifice.

2.1.2. *The duty to perform fixed and occasional sacrifices* In the case of fixed and occasional sacrifices, Mīmāṃsakas since the time of Jaimini maintain that the non-performance of the sacrifice is a *fault* (*doṣa*).[7] It seems, then, that a fixed sacrifice is strictly obligatory, in the sense that *an eligible performer of the sacrifice omits its performance at his peril*.

This can be taken to mean that negative consequences attend on the non-performance of a fixed sacrifice:

> Indeed, this person who, though being a performer of the full- and new-moon sacrifices, omits either the full-moon or the new-moon sacrifice, is cut off from heaven.[8]

In this case, however, the negative consequences could be interpreted in two slightly different ways: either simply as the absence of a desired result (which is explicitly stated in this case to be heaven, i.e. happiness[9]), or as something that is negative in itself (which may be implied, supposing that the absence of happiness implies the presence of pain, grief, suffering, and so on). The second case would mean that omitting the performance of a fixed or occasional sacrifice would lead to a sanction. Prior to the new phase of Mīmāṃsā associated with Kumārila, Prabhākara and Maṇḍana in the sixth to eighth centuries, there seems to have been very little reflection on the difference between 'absence of happiness' and 'presence of unhappiness'.[10]

2.1.3. *General features of sacrifices: results* (phala) *and eligibility* (adhikāra) According to Mīmāṃsā authors, every sacrifice is said to have some result (*phala*):[11]

- for elective sacrifices, this principle is straightforward, since they are defined by an orientation towards a specific desired result;
- for fixed (and occasional) sacrifices, Mīmāṃsakas maintain that the result is 'heaven' (*svarga*), which they define as felicity (*prīti*). The desire for heaven is

[7] *tadakarmaṇi ca doṣas tasmāt tato viśeṣaḥ syāt pradhānenābhisaṃbandhāt* (*Exegetic Aphorisms* 6.3.3), see fn. 15.

[8] Śabara quotes this passage from a Vedic texts, namely *Taittirīya Saṃhitā* 2.2.5.4, in his commentary on *Exegetic Aphorisms* 6.3.3.

[9] Heaven is explicitly equated to happiness, e.g. in *Commentary on Exegetic Aphorisms* 6.1.1 (*prītisādhane svargaśabda iti*) and 6.1.2 (*prītiḥ svarga iti*).

[10] However, one might try to locate some indirect hints already in Śabara's discussions of such sacrifices. For instance, in his commentary on *Exegetic Aphorisms* 2.4. *adhikaraṇa* 1, he discussed the issue of whether the duty to perform the *agnihotra* sacrifice 'as long as one is alive' means that there is a single performance of the *agnihotra*, repeated through every day of one's life, or whether an *agnihotra* is completed every single day and then performed again on the next day. The two alternatives have some bearing on the issue of whether omitting the *agnihotra* entails the lack of a positive result or an additional sanction. In fact, if there is only a single performance of the *agnihotra*, lasting throughout one's whole life, happiness as its result can only come at a successive time, i.e. in the next life, and the absence of this result cannot be the reason for one's unhappiness during one's life. If, by contrast, each *agnihotra* is completed on a given day and delivers its result on a daily basis, then this could be enough to motivate one to refrain from omitting even a single performance.

[11] Early Mīmāṃsā authors mention the opinion of an inner-Mīmāṃsā opponent, called Bādari (of whom no works are extant) who claims that the result plays no role at all in sacrifices, but this point of view is explicitly refuted by all later authors. *Natarajan 1995* maintains that there is a split among Mīmāṃsakas regarding the connection of each of these types of sacrifice with a result, but the position she ascribes to Prabhākara, namely, that there is no result for fixed and occasional sacrifices, seems closer to the position of Bādari than to the one of Prabhākara. If we exclude Bādari, then all Mīmāṃsakas agree that every sacrifice leads to a result.

always present in all beings and the so-called 'Viśvajit Principle' guarantees that heaven is the result of any sacrifice in which no other result is explicitly mentioned.

Secondly, every sacrifice is characterized by an *eligibility* (*adhikāra*). The eligibility identifies the person who will possess the result of the sacrifice. This is particularly relevant in the context of Vedic sacrifices, since they always include many performers. The *adhikārin* is the one who decides to perform the sacrifice (and pays several Brāhmaṇas to perform it) and the sacrifice's result will accrue to him only. Thus, being eligible means being both enjoined to perform the sacrifice and entitled to its result; the former implies that one can actually hear a Vedic injunction and understand oneself as the addressee, which, in general, implies that one belongs to one of the social groups that is traditionally expected to study the Vedas. Being eligible further implies having enough material wealth to organize the performance of a sacrifice: eligibility thus always includes ability, and so it incorporates a version of the 'ought entails can' thesis, according to which an injunction to perform a certain sacrifice applies only to a person that is able, in principle, to complete the performance of that sacrifice. For instance, a lame person who desires cattle and hears the Vedic injunction 'The one who desires cattle should sacrifice with the *citrā* sacrifice' is, notwithstanding her desires, not the addressee of the injunction because she could not be able to carry out the sacrificial actions required. The negative consequences that, as noted above (Section 2.1.2), would generally follow from non-performance of a sacrifice only apply to an eligibile sacricer's non-performance[12] and negative consequences could follow if she does perform it. By contrast, a person who has the physical (see *Exegetic Aphorisms* 6.1.42) and economical ability (see *Exegetic Aphorisms* 6.7.18–20) to perform a given sacrifice and desires its output is the addressee of the relevant injunction, independently of her decision to carry it out or not. Accordingly, eligibility applies to a person for a longer period of time than will, desire, or inclination and is defined, partly, by the wherewithal needed to successfully perform a given sacrifice. Eligibility is not about possessing a given ability temporarily, but about one's overall condition as a human being. For instance, if a blind person were to receive once in a month and for one day only a digital visor enabling her to see, she would still lack the *adhikāra* for performing a sacrifice lasting longer than the day in which she can see. A person who should receive a huge amount of money for only one day would not qualify as wealthy enough to perform a complex sacrifice and so on. Symmetrically, if one were to temporarily lose one's ability to see (e.g. because one has looked directly into the sun), or if one had run out of a given ritual substance, one would not lose one's *adhikāra*—the loss of *adhikāra* is sometimes discussed in the texts, but only as a consequence of some traumatic experience (e.g. a major violation of dharma rules).

2.1.4. *The duty to perform elective sacrifices* What makes a sacrifice 'elective' rather than fixed or occasional is the presence of a desire for a specific result (as distinguished from the desire for happiness, which is always present) and the fact that there are no negative consequences in case one does not perform it, apart from not getting the intended result. Furthermore, the deontic strength of injunctions to perform elective sacrifices seems weaker: for example, there is an injunction that says 'one who desires cattle should perform the *citrā* sacrifice.' Suppose that you desire cattle and satisfy the eligibility requirement:

[12] It is perhaps worth noting that this is not necessarily the case with all obligations in all cultures. One is, for instance, immediately reminded of the contrast between law and grace in Paul, or of the paradox of the 'infinite responsibility' discussed by Emmanuel Levinas. In an Indian context, one may evoke the Viśiṣṭādvaita Vedānta analysis of one's helplessness in regard to moral laws which could never be fulfilled.

are you under an *obligation* to perform the *citrā* sacrifice? Most Mīmāṃsakas would say that you are not, i.e. that even one who desires the specified result can omit the performance of the sacrifice without any adverse consequences, apart from the absence of the desired result. This could mean that fixed and occasional sacrifices differ from elective sacrifices in the fact that their result is something one cannot live without (i.e. happiness), so that its absence is the *summum malum*.

Under these circumstances, it is unclear whether we should speak of an 'obligation' to perform elective sacrifices at all. On the one hand, there is a strong presumption in Mīmāṃsā that *all Vedic injunctions operate in the same way*, although there is considerable debate about how they operate. This means, in particular, that in the Vedas injunctions to perform elective sacrifices have the same *linguistic presentation* as injunctions to perform fixed and occasional sacrifices. Thus, if we interpreted injunctions to perform fixed and occasional sacrifices as obligations relying only on their linguistic presentation, we should, in principle, do the same with injunctions to perform elective sacrifices. On the other hand, in Euro-American deontic theories, obligations are often considered as 'duals' of permissions, so that the performance of an action is obligatory if and only if its non-performance is not permitted.[13] However, we have seen that the non-performance of elective sacrifices is permitted, though not recommended. Thus, it seems plausible to conceive of injunctions to perform elective sacrifices as *recommendations*.

A further complication is that once one has undertaken the performance of an elective sacrifice, then she has a duty to carry out the sacrifice completely. Mīmāṃsā authors say in this connection that an elective sacrifice is *undertaken* because of a desire; but once it has been undertaken, the undertaking itself (*ārambha*) serves as the occasion (*nimitta*) for the *completion* of the elective sacrifice. Śabara suggests (in *Commentary on Exegetic Aphorisms* 6.1.15) that there is also a social sanction if you start performing an elective sacrifice without being able to complete it, in the sense that people will think less of you.

2.1.5. *The 'good enough' principle (*yathāśakti-nyāya*)* Regarding the *subsidiaries* of *fixed* sacrifices, but not of *elective* sacrifices, it is said that one must perform them 'insofar as one is able' (*yathāśakti*).[14] If you undertake a fixed sacrifice like the *agnihotra* sacrifice, then you do not have a duty to 'dot the *i*s and cross the *t*s'. That is to say, if the Vedas enjoin certain actions as subsidiaries of the *agnihotra*, and you are not in a position to perform those actions, then you may simply omit them and the injunction to perform the *agnihotra* will nevertheless be taken as fulfilled.[15] This represents what we can call the *'good enough' principle*.

A few more words are needed in order to show how the 'good enough' principle relates to the notion of eligibility (*adhikāra*, see Section 2.1.3). As we said, eligibility postulates that one must in theory be able to perform a sacrifice in order to be the addressee of the relevant injunction. If one is *a priori* unable to perform the sacrifice, for instance, because one cannot walk or cannot afford the purchase of the sacrificial substances, one is not the addressee of the sacrificial prescriptions and is therefore not under their obligations. By contrast, if one is *a priori* able to perform the sacrifice but concretely unable to perform a

[13] See *Mabbott 1966* for a concise survey of traditional deontic theories in Euro-American philosophy and a discussion of the various notions of 'duty' employed in these theories.

[14] See *Exegetic Aphorisms* 6.3, *adhikaraṇa* 1.

[15] *Exegetic Aphorisms* 6.3, *adhikaraṇa* 1. See especially *Exegetic Aphorisms* 6.3.3 (*tadakarmaṇi ca doṣas tasmāt tato viśeṣaḥ syāt pradhānenābhisaṃbandhāt* 'If you do not perform it (the principal sacrifice) there is a fault, therefore there is a distinction between the [principal sacrifice] [and the auxiliaries], since the [fault] is mentioned only in connection with the principal sacrifice').

specific subsidiary act (for instance, because one does not have a given sacrificial substance available), one can perform the whole sacrifice according to the 'good enough' principle without violating her obligation.

How should we understand, exactly, the distinction between a priori impossibility and concrete impossibility? It is *not* a distinction between a deficit regarding the person and a deficit regarding her wealth (so that one could say that the *adhikāra* regards the person and the *yathāśaktinyāya* her possessions) for a person who cannot have enough substances for the performance of a sacrifice (e.g. because she cannot own wealth) lacks the relevant *adhikāra* (see *Exegetic Aphorisms*, chapter 6.1, see also, Section 2.1.3). Rather, the distinction involves a general possibility and a concrete condition. A person who is in general physically and economically able to perform a sacrifice has the relevant *adhikāra*. Her lacking a concrete sacrificial ingredient at a given moment of time, while nothing precludes that she might acquire it at a later moment of time, does not hinder the fact that she has the *adhikāra*.

In contrast, regarding *elective* sacrifices, it is said that one must perform them by completing all their subsidiaries exactly as prescribed in the Vedas.[16] In other words, if one does not perform the subsidiary actions of a given sacrifice precisely as they are enjoined in the Vedic text, her performance does not count as a satisfactory performance of the enjoined sacrifice.

2.1.6. *Discussion* Let us review what we have seen so far, which represents the system of Common Mīmāṃsā: The standard classification of sacrifices distinguishes three *principal* types, namely, fixed, occasional, and elective. We can *abstract* at least two kinds of deontic properties that apply to primary actions:

- obligation: You omit the enjoined action at your peril. This applies to fixed and occasional sacrifices.
- recommendation: You may omit the enjoined action without any adverse consequences. This applies to elective sacrifices, but *only before one has started them*; if one has undertaken an elective sacrifice, one fails to *complete* it at one's peril.

Furthermore, we can distinguish between two ways in which one has to perform subsidiary actions:

- 'exactly as prescribed in the texts': this applies to the subsidiaries of elective sacrifices and means that their performance does not count unless it is done precisely as the Vedas prescribe.
- 'as much as possible': this applies to the subsidiaries of fixed and occasional sacrifices and means that their performance counts even if it is not done precisely as the Vedas prescribe, as long as it is done to the best of the sacrificer's ability.

In a sense, there is a kind of *deontic reversal* when moving from primary actions (i.e. sacrifices) to their subsidiaries. If a sacrifice is *obligatory*, then its subsidiaries are to be performed *as much as one can* (insofar as the 'good enough' principle applies) and so it is permitted that some of them are not performed or are simplified. By contrast, if a sacrifice is *recommended*, then its subsidiaries are to be performed *exactly as prescribed* (insofar as the 'good enough' principle does not apply), so it is not permitted to simplify their performance.

16 See *Exegetic Aphorisms* 6.3, *adhikaraṇa* 2.

3. Formalizing the Mīmāṃsa theory

3.1. The system $\mathbf{S_0}$

The distinction among types of Vedic duties discussed by Mīmāṃsa authors and illustrated in Section 2.1 relies on the interplay between certain deontic notions and certain types of circumstance; in this section we move from an informal to a formal setting and provide a logical analysis of the Mīmāṃsa theory. We start by observing that standard languages of *propositional logic* are not adequate to capture basic differences among ritual actions, since this requires the possibility of analyzing the internal structure of propositions. For instance, in standard propositional languages it is not possible to define, *in general terms*, the properties of sacrifices that should be performed in circumstances of different kind (e.g. when a relevant occasion takes place or a relevant desire arises) or the relation between a sacrifice and its subsidiary actions. Here we propose to adopt a slightly richer language, hereafter simply called $\mathcal{L}$, which *simulates* some aspects of first-order languages while remaining closely related to propositional ones, since it does not involve any form of quantification. Our choice is motivated by the aim of keeping the logical analysis as simple as possible.

The language $\mathcal{L}$ is built *ad hoc* to represent the theory of Vedic duties in early Mīmāṃsa and includes four categories of *individual constants*: sacrifice constants (which denote entities like 'the *citrā* sacrifice' and 'the full-moon sacrifice') action constants (which denote entities like 'grinding the rice' and 'sifting the flour'), outcome constants (which denote entities like 'cattle' and 'happiness') and event constants (which denote entities like 'the birth of a son' and 'a solar eclipse'). These constants are used to build atomic formulas of some specified kind, thanks to the presence of a small set of *predicate symbols*, which express fundamental notions in the Mīmāṃsa analysis. Furthermore, it includes a *propositional constant* associated with the notion of eligibility, *boolean operators* and *intensional operators* for strong/weak duties. The full list of primitive symbols in $\mathcal{L}$ is the following:

- a countable set of individual constants for sacrifices (*sacrifice constants*) $SAC = \{s_1, s_2, s_3, \ldots\}$;
- a countable set of individual constants for actions (*action constants*) $ACT = \{a_1, a_2, a_3, \ldots\}$;
- a countable set of individual constants for events (*event constants*) $EVT = \{e_1, e_2, e_3, \ldots\}$;
- a countable set of individual constants for outcomes (*outcome constants*) $OUT = \{o_1, o_2, o_3, \ldots\}$;
- a propositional constant (*eligibility constant*) $\mathfrak{e}$;
- the boolean connectives $\neg$ (*negation*) and $\rightarrow$ (*material implication*);
- the intensional operators $\mathcal{O}$ (*strong duty*, or obligation) and $\mathcal{R}$ (*weak duty*, or recommendation);
- the unary predicate symbol *und* (for 'is undertaken') applied to elements of the set *SAC*;
- the unary predicate symbols *eap* (for 'is performed exactly as prescribed', *yathānyāya*) and *amp* (for 'is performed as much as possible', *yathāśakti*) applied to elements of the set *ACT* (see Section 2.1.5 for a discussion of the corresponding Sanskrit terms);
- the unary predicate symbol *des* (for '. . . is desired') applied to elements of *OUT*;
- the unary predicate symbol *tpl* (for '. . . takes place') applied to elements of *EVT*;

- the binary predicate symbol *sub* (for '... is a subsidiary of ...') applied to elements of the set $ACT \times SAC$ (i.e. the cartesian product of *ACT* and *SAC*);[17]
- round brackets.

The exhaustive meaning of these symbols will be clarified below. The boolean connectives $\wedge$ (*conjunction*), $\vee$ (*inclusive disjunction*) and $\equiv$ (*material equivalence*), are defined in the usual way in terms of the primitive ones. It is convenient to consider also the set of *propositional atoms* in $\mathcal{L}$, *ATOM*, which is the smallest set described by the following clauses:

- for any $s_i \in SAC$, $und(s_i) \in ATOM$;
- for any $a_i \in ACT$, $eap(a_i), amp(a_i) \in ATOM$;
- for any $e_i \in EVT$, $tpl(e_i) \in ATOM$;
- for any $o_i \in OUT$, $des(o_i) \in ATOM$;
- for any $a_i \in ACT$ and $s_j \in SAC$, $sub(a_i, s_j) \in ATOM$;
- $\mathfrak{e} \in ATOM$.

An arbitrary member of *ATOM* will be denoted by *at*. Propositional atoms have to be read as follows: $und(s_i)$ means that the sacrifice described by s_i (e.g. the full-moon sacrifice) is undertaken; $eap(a_i)$ that the action described by a_i (e.g. grinding the rice) is carried out exactly as prescribed and $amp(a_i)$ that the same action is performed as much as possible (i.e. according to one's possibility); $tpl(e_i)$ means that the event described by e_i (e.g. the full moon) takes place and $des(o_i)$ means that the outcome described by o_i (e.g. cattle) is desired; $sub(a_i, s_j)$ means that the action described by a_i is a subsidiary of the sacrifice described by s_j (e.g. offering a rice-cake to Agni is a subsidiary of the full-moon sacrifice). Finally, we can read the propositional constant $\mathfrak{e}$ as 'minimal requirements to perform sacrifices are satisfied', so this constant represents *eligibility*.[18] The set of *well-formed formulas* (hereafter, *wffs*) of $\mathcal{L}$ is the smallest set satisfying the clauses below:[19]

- *at* is a wff, for any $at \in ATOM$;
- if ϕ is a wff, then so are $\neg(\phi)$, $\mathcal{O}(\phi)$ and $\mathcal{R}(\phi)$;
- if ϕ and ψ are wffs, then so is $(\phi) \rightarrow (\psi)$.

Round brackets that should apply to the arguments of boolean and intensional operators are omitted in formulas if there is no risk of ambiguity; for instance, $(\phi) \rightarrow (\psi)$ can be rewritten as $\phi \rightarrow \psi$. We use the notion of *main operator* in a formula in the usual way; for instance, in a formula of kind $\phi \rightarrow (\psi \rightarrow \chi)$ we say that the first occurrence of $\rightarrow$ is the

[17] This means that the binary predicate symbol *sub* only defines the relation linking a subsidiary to a main sacrifice. It does not define the relation holding between a subsidiary and its own further subsidiaries (e.g. the relation holding between 'offering a rice-cake' and 'grinding the rice' or 'baking the cake'). We decided to focus on the first distinction since this alone has consequences for the categorization of sacrifices. This is also the reason why we introduced a special category for sacrifices, namely *SAC*.

[18] See the discussion on the notion of eligibility in Section 2.1.3. For the use of propositional constants in a deontic setting, see *Anderson 1956* and *Åqvist 1987*. Propositional constants allow one to codify in a formal language certain states-of-affairs which are particularly relevant for deontic reasoning; in our case, the notion of eligibility. Indeed, we will see that the constant $\mathfrak{e}$ restricts the set of situations in which certain ritual actions have to be performed.

[19] In a sense, the language $\mathcal{L}$ captures two different intuitions that are often compared in deontic logic. One intuition, originated with *von Wright 1951*, is that actions have a special status in deontic reasoning; this intuition, which leads to an analysis of what one 'ought to do' is here captured by making explicit reference to actions via individual constants. The other intuition, originated with *Anderson 1956*, is that sometimes deontic reasoning involves relations among states-of-affairs (e.g. 'if there are criminals, there ought to be sanctions'); this intuition, which leads to an analysis of what 'ought to be the case', is here captured by the possibility of applying boolean and intensional operators to arbitrary formulas, rather than formulas involving actions alone.

main material implication. We propose to read $\mathcal{O}\phi$ as 'in every situation in which *strong* Vedic duties are observed ϕ is the case' and $\mathcal{R}\phi$ as 'in every situation in which *weak* Vedic duties are observed ϕ is the case'. The difference between strong and weak Vedic duties resembles the difference between *obligations* and *recommendations*, as it was pointed out in Sections 2.1.2 and 2.1.4; thus, another possible reading of $\mathcal{O}\phi$ and $\mathcal{R}\phi$ is, respectively, 'it is obligatory, according to the Vedas, that ϕ' and 'it is recommended, according to the Vedas, that ϕ'.

We stipulate the following definitions (for $s_i \in SAC$), which capture the main properties of the three types of sacrifice:

– $fixed(s_i) =_{def} \mathcal{O}(\mathfrak{e} \rightarrow und(s_i))$;
– $occasional(s_i)/e_n =_{def} \mathcal{O}((\mathfrak{e} \wedge tpl(e_n)) \rightarrow und(s_i))$;
– $elective(s_i)/o_n =_{def} \mathcal{R}((\mathfrak{e} \wedge des(o_n)) \rightarrow und(s_i))$.

Thus, a sacrifice is fixed if and only if (hereafter, iff) it is obligatory to undertake it in any circumstance in which eligibility is met; a sacrifice is occasional iff it is obligatory to undertake it whenever eligibility is met and the relevant occasion takes place; finally, a sacrifice is elective iff it is recommended to undertake it whenever eligibility is met and the relevant desire arises.

We will now build a system S_0 over the language $\mathcal{L}$ in order to be able to represent generic patterns of reasoning in terms of our formalization of the Mīmāṃsa theory of Vedic injunctions. We will proceed in a series of steps, illustrating some plausible principles that can be taken into account in the axiomatic basis of S_0. First of all, we want S_0 to be an extension of the *classical propositional calculus*,[20] so we assume what follows (where the expression $\vdash_{S_0} \phi$ means that ϕ is a theorem of S_0 and the expression $\phi_1, \ldots, \phi_n \vdash_{S_0} \phi_{n+1}$ means that ϕ_{n+1} is derivable in S_0 from the list of assumptions $\phi_1, \ldots, \phi_n$):

$$\vdash_{S_0} \phi, \text{ for any propositional tautology } \phi \quad (1)$$

$$\phi, \phi \rightarrow \psi \vdash_{S_0} \psi \quad (2)$$

Then, we add a series of *axiom-schemata* (hereafter, also called axioms or schemata, for the sake of brevity) concerning the intuitive relation between some predicate symbols in $\mathcal{L}$.[21] These axiom-schemata allow us to make universal claims with reference to certain classes of individual constants. The first schema (where a_i is any constant in ACT) concerns the relation between *eap* and *amp*:

$$eap(a_i) \rightarrow amp(a_i). \quad (3)$$

Axiom-schema (3) means that whenever an action is completed, then that action is also performed as much as possible. We also need some principles concerning the interaction between fixed, occasional, elective sacrifices and their subsidiaries. Consider the following ones:

$$(fixed(s_i) \wedge sub(a_j, s_i)) \rightarrow \mathcal{O}(\mathfrak{e} \rightarrow (und(s_i) \rightarrow amp(a_j))), \quad (4)$$

20 The legitimacy of this move is explained in *Freschi et al. 2017*. In fact, a Mīmāṃsā embedded in Jayanta's *Nyāyamañjarī* explains how, when a contradiction is considered, denying one alternative makes the other true. This implies the legitimacy of *reductio ad absurdum*, thus justifying the use of classical logic.

21 In our presentation of axiom-schemata there is some abuse of notation; for instance, in the case of (3) we should employ a symbol different from a_i, which would stand for an arbitrary element of *ACT* (namely a metalinguistic symbol); however, in our opinion the notation adopted here is more accessible for non-logicians and we will try to avoid any risk of confusion by supporting the presentation of various axiom-schemata with some informal comments.

$$(occasional(s_i)/e_n \wedge sub(a_j, s_i)) \rightarrow \mathcal{O}((\mathfrak{e} \wedge tpl(e_n)) \rightarrow (und(s_i) \rightarrow amp(a_j))), \quad (5)$$

$$(elective(s_i)/o_n \wedge sub(a_j, s_i)) \rightarrow \mathcal{O}((\mathfrak{e} \wedge des(o_n)) \rightarrow (und(s_i) \rightarrow eap(a_j))). \quad (6)$$

Axiom-schema (4) says that, obligatorily, whenever eligibility is met ($\mathfrak{e}$), if a fixed sacrifice (s_i) is undertaken, then any of its subsidiaries (a_j) is performed as much as possible. Axiom-schema (5) says that, obligatorily, whenever eligibility is met ($\mathfrak{e}$) and a certain occasion (e_n) takes place, if a relevant occasional sacrifice (s_i) is undertaken, then any of its subsidiaries (a_j) is performed as much as possible. Axiom-schema (6) says that, obligatorily, whenever eligibility is met ($\mathfrak{e}$) and a certain outcome (o_n) is desired, if a relevant elective sacrifice (s_i) is undertaken, then any of its subsidiaries (a_j) is performed exactly as prescribed. These axioms represent the 'deontic reversal' mentioned in Section 2.1.6: on the one hand, fixed sacrifices represent duties in a strong sense (or obligations, $\mathcal{O}$), while their subsidiaries have to be performed only as much as possible (*amp*); on the other hand, elective sacrifices represent duties in a weak sense (or recommendations, $\mathcal{R}$), while their subsidiaries, once the main action is undertaken, have to be carried out precisely as prescribed (*eap*).

Furthermore, we need to add axioms for our operators $\mathcal{O}$ and $\mathcal{R}$. In the case of $\mathcal{O}$, a possible solution consists in extracting the intended axioms and rules from a preliminary system to represent deontic reasoning in Mīmāṃsa, developed in *Ciabattoni et al. 2015*; such system, called **bMDL** (basic Mīmāṃsa Deontic Logic), is based on a propositional language with both alethic and deontic modalities. The result of this 'extraction procedure' is the following list of principles (see the Appendix for the detailed proof and for an explanation of the sense in which **bMDL** can be said to be a system inspired by the Mīmāṃsa school):

$$\frac{\phi \equiv \psi}{\mathcal{O}\phi \equiv \mathcal{O}\psi}, \quad (7)$$

$$\mathcal{O}(\phi \wedge \psi) \rightarrow (\mathcal{O}\phi \wedge \mathcal{O}\psi), \quad (8)$$

$$\neg(\mathcal{O}\phi \wedge \mathcal{O}\neg\phi). \quad (9)$$

The principle (7) says that if the states-of-affairs described by ϕ and ψ are logically equivalent, then either both or neither ought to be the case (more precisely, according to our specific reading of $\mathcal{O}$, either both or neither hold in all situations in which strong Vedic duties are observed). The principle (8) says that if the states-of-affairs described by ϕ and ψ jointly form a state-of-affairs which ought to be the case, then each of them ought to be the case. Finally, (9) says that it is not the case that two conflicting states-of-affairs ought to be the case.[22]

In the case of $\mathcal{R}$, instead, a different set of logical principles is required; in particular $\neg(\mathcal{R}\phi \wedge \mathcal{R}\neg\phi)$ seems too strong, since $\mathcal{R}$ expresses a deontic property which is closer to recommendation than obligation and there are examples of *conflicting* recommendations in the Vedas, such as $\mathcal{R}\phi$ and $\mathcal{R}\psi$, where ψ entails $\neg\phi$. For instance, there are two different Vedic prescriptions concerning the desire of rain. One prescription says that if a person desires rain, he or she should perform the *kāriri* sacrifice; the other prescription says that if a person desires rain, he or she should perform the *twelve-nights* sacrifice (*Kauśika Sūtra* 5.5.41.1); however, a person cannot perform both sacrifices at the same time (and probably would not perform them in succession, given that one of the two sacrifices is enough to get the intended result). Accordingly, we propose to

[22] Here the term 'situation' is used in an informal way. In Section 3.2 a situation will represent a completely determined and exhaustive set of states-of-affairs, namely what is usually called a (possible) 'world' in the literature on intensional logic. For further philosophical analysis of the relation between worlds and states-of-affairs, see *Adams 1974*.

replace $\neg(\mathcal{R}\phi \wedge \mathcal{R}\neg\phi)$ with $\neg\mathcal{R}(\phi \wedge \neg\phi)$, which guarantees, at least, that there are no *self-contradictory* recommendations. Thus, we take the following set of principles for $\mathcal{R}$:

$$\frac{\phi \equiv \psi}{\mathcal{R}\phi \equiv \mathcal{R}\psi}, \tag{10}$$

$$\mathcal{R}(\phi \wedge \psi) \rightarrow (\mathcal{R}\phi \wedge \mathcal{R}\psi), \tag{11}$$

$$\neg\mathcal{R}(\phi \wedge \neg\phi). \tag{12}$$

The reading of (10), (11) and (12) can be inferred from our considerations above and their relations with (7), (8) and (9). The axiomatic basis of the system S_0 is finally defined by the list of principles (1)–(12).

Notice that in S_0 we do not impose any connection between $\mathcal{O}$ and $\mathcal{R}$, since, according to our reading of the two operators, $\mathcal{O}\phi$ simply means that ϕ represents a strong duty in the Vedas and $\mathcal{R}\phi$ that ϕ represents a weak duty in the Vedas. On the one hand, relying on the parallelism mentioned above between strong duties and obligations and on the one between weak duties and recommendations (which inspired a simpler reading for $\mathcal{O}$ and $\mathcal{R}$), one could add the further principle that whatever is obligatory is also recommended:

$$\mathcal{O}\phi \rightarrow \mathcal{R}\phi. \tag{13}$$

On the other hand, sometimes Mīmāṃsā authors encountering Vedic statements which seem to prescribe as an elective sacrifice what is already known to be a fixed sacrifice read these statements as prescribing a different sacrifice, bearing by chance the same name as the fixed sacrifice (see *Exegetic Aphorisms* 2.3.24–25). This leads to a completely different idea of the relation between strong duties and weak duties, namely that they are mutually exclusive; we can formalize this idea by means of the principle below:

$$\mathcal{O}\phi \rightarrow \neg\mathcal{R}\phi. \tag{14}$$

In what follows we will focus on the semantic characterization of the basic system S_0, in which neither of the schemata (13) and (14) is taken into account. However, if we call S_1 and S_2 the systems obtained by respectively adding (13) and (14) to the axiomatic basis of S_0, we will see that both S_1 and S_2 are *consistent* systems.

3.2. *Semantic characterization of* $\mathbf{S_0}$

We introduce now a formal semantics for the system S_0. We interpret the language $\mathcal{L}$ in *neighborhood frames*, which are here structures of kind $\mathfrak{F} = \langle W, W_{\mathfrak{e}}, N^+, N^- \rangle$. $W = \{w_1, w_2, w_3, \ldots\}$ (also $w, v, u, \ldots$) is a set of possible *situations* (or *worlds*) and $W_{\mathfrak{e}} \subseteq W$ is the set of situations in which the eligibilty conditions are met. $N^+ : W \longrightarrow (\wp(\wp(W))$ is said to be a *neighborhood function*; for any $w \in W$, $N^+(w) \subseteq \wp(W)$ is said to be the N^+*-sphere* of w. Similar definitions can be used for N^-. Informally, $N^+(w)$ and $N^-(w)$ can be respectively taken as representing the set of strong Vedic duties (obligations) and the set of weak Vedic duties (recommendations) which apply to the situation w.

A model over a neighborhood frame, also called *neighborhood model*, is a structure of kind $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$, where $\mathfrak{F}$ is the underlying frame and $V : ATOM \longrightarrow \wp(W)$ is said to be a *valuation function*. For any $at \in ATOM$, $V(at) \subseteq W$ is said to be the valuation of at in $\mathfrak{M}$. The valuation function satisfies the minimal property $V(\mathfrak{e}) = W_{\mathfrak{e}}$, which ensures that in every model the set of situations in which eligibility is met corresponds to the set $W_{\mathfrak{e}}$. Models can be denoted also by pointing out elements of the underlying frame, as in $\mathfrak{M} = \langle W, W_{\mathfrak{e}}, N^+, N^-, V \rangle$.[23] Since in the language $\mathcal{L}$ quantification over individuals

[23] For a presentation of standard neighborhood frames and models, see *Chellas 1980*.

is not available and individual constants are interpreted *in the same way across worlds* and models, we *do not* need to add domains of individual entities to our frames and models. This fact provides a semantic justification to our claim that $\mathcal{L}$ remains closely related to propositional languages (while allowing one to analyze the internal structure of propositions).

Given a model $\mathfrak{M}$, we say that *all strong* (respectively, *weak*) *Vedic duties are observed in a situation w* iff $w \in X$ whenever $X \in N^+(w)$ (respectively, $X \in N^-(w)$). Indeed, this corresponds with the fact that w verifies all instances of the schema $\mathcal{O}\phi \rightarrow \phi$ (respectively, $\mathcal{R}\phi \rightarrow \phi$). Notice that, in principle, different situations can be associated with different duties.

Formulas of $\mathcal{L}$ are evaluated with reference to a world in a model. Truth-conditions for formulas are defined below, where $\mathfrak{M}$ and w are, respectively, an arbitrary model and an arbitrary world in it (given a formula $\phi \in \mathcal{L}$, the expression $\mathfrak{M}, w \vDash \phi$ means that ϕ is true at w in $\mathfrak{M}$, whereas the expression $\mathfrak{M}, w \nvDash \phi$ means that ϕ is false at w in $\mathfrak{M}$):

- for any $at \in ATOM$, $\mathfrak{M}, w \vDash at$ iff $w \in V(at)$;
- $\mathfrak{M}, w \vDash \neg\phi$ iff $\mathfrak{M}, w \nvDash \phi$;
- $\mathfrak{M}, w \vDash \phi \rightarrow \psi$ iff either $\mathfrak{M}, w \nvDash \phi$ or $\mathfrak{M}, w \vDash \psi$;
- $\mathfrak{M}, w \vDash \mathcal{O}\phi$ iff $\|\phi\|^{\mathfrak{M}} \in N^+(w)$, where $\|\phi\|^{\mathfrak{M}} = \{v \in W : \mathfrak{M}, v \vDash \phi\}$;
- $\mathfrak{M}, w \vDash \mathcal{R}\phi$ iff $\|\phi\|^{\mathfrak{M}} \in N^-(w)$, where $\|\phi\|^{\mathfrak{M}} = \{v \in W : \mathfrak{M}, v \vDash \phi\}$.

It is interesting to explore the intuition behind the last two clauses in the list. Indeed, relying on a convention about the interpretation of intensional operators in standard neighborhood models, we can say that in a model any injunction corresponds to the set of situations in which it is fulfilled (notice that this convention is grounded on our reading of $\mathcal{O}\phi$ and $\mathcal{R}\phi$ as 'in all situations in which strong Vedic duties are observed, ϕ is the case' and 'in all situations in which weak Vedic duties are observed, ϕ is the case').

A formula ϕ is *valid* in a model $\mathfrak{M}$ iff for all $w \in W$, we have $\mathfrak{M}, w \vDash \phi$; we represent this fact also as $\mathfrak{M} \vDash \phi$. A formula ϕ is valid in a class of models C_m iff, for every $\mathfrak{M}_i \in C_m$, we have $\mathfrak{M}_i \vDash \phi$; we represent this fact also as $C_m \vDash \phi$. A system X built over the language $\mathcal{L}$ is *sound* with respect to (hereafter, w.r.t.) a class of models C_m iff every formula which is a theorem of X is valid in C_m; X is *complete* w.r.t. C_m iff every formula which is valid in C_m is a theorem of X; X is *characterized* by a class of models C_m iff it is both sound and complete w.r.t. C_m. If a system X is sound w.r.t. a class of models C_m, we also say that C_m is a class of models *for X*. We can similarly speak of soundness, completeness and characterization of a system X w.r.t. a single model $\mathfrak{M}$. Finally, a system X is *consistent* iff there exists some model for X.

We introduce now a specific class of models which will be used for the semantic characterization of S_0. Let K_m be the class of neighborhood models $\mathfrak{M} = \langle W, W_{\mathfrak{e}}, N^+, N^-, V\rangle$ satisfying the following properties:

P1 for any $X, Y \in \wp(W)$ s.t. $X \subseteq Y$ and any $w \in W$, if $X \in N^+(w)$, then $Y \in N^+(w)$;
P2 for any $X, Y \in \wp(W)$ s.t. $X \subseteq Y$ and any $w \in W$, if $X \in N^-(w)$, then $Y \in N^-(w)$;
P3 for any $X \in \wp(W)$ and any $w \in W$, if $X \in N^+(w)$, then $\overline{X} = \{v \in W : v \notin X\} \notin N^+(w)$;
P4 for any $w \in W$, $\emptyset \notin N^-(w)$;
P5 for any $a_i \in ACT$, $V(eap(a_i)) \subseteq V(amp(a_i))$;
P6 for any $a_j \in ACT$, $s_i \in SAC$ and $w \in W$, if $w \in V(sub(a_j, s_i))$ and $\|\mathfrak{e} \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^+(w)$, then $\|\mathfrak{e} \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}} \in N^+(w)$;

P7 for any $a_j \in ACT$, $s_i \in SAC$ and $w \in W$, if $w \in V(sub(a_j, s_i))$ and $\|(\mathfrak{e} \wedge tpl(e_n)) \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^+(w)$, for some $e_n \in EVT$, then $\|(\mathfrak{e} \wedge tpl(e_n)) \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}} \in N^+(w)$;

P8 for any $a_j \in ACT$, $s_i \in SAC$ and $w \in W$, if $w \in V(sub(a_j, s_i))$ and $\|(\mathfrak{e} \wedge des(o_n)) \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^-(w)$, for some $o_n \in OUT$, then $\|(\mathfrak{e} \wedge des(o_n)) \rightarrow (und(s_i) \rightarrow eap(a_j))\|^{\mathfrak{M}} \in N^+(w)$.

We will prove that the system S_0 is sound and complete w.r.t. the class K_m. First, we show that K_m is a non-empty class, namely that there is at least one model satisfying properties P1-P8. We provide a model based on the argument illustrating the duty to perform the *citrā* sacrifice.

Let us consider the Mīmāṃsā analysis of the *citrā* sacrifice: The *citrā* sacrifice is an elective sacrifice prescribed for people who desire cattle. One who is eligible (i.e. because he has studied the Vedas, is physically able to perform a sacrifice and possesses enough wealth to perform it) and desires cattle should perform the *citrā* sacrifice. Since it is an elective sacrifice, one needs to carry out its subsidiaries, such as the offerings of a cake on eight cups to Agni, of a *caru*[24] on eleven cups to Soma, of a cake on eight cups to Tvaṣṭṛ, of a *caru* on eight cups to Sarasvatī, of a *caru* on eight cups to Sarasvant, of a *caru* to Sinīvalī and of a cake on eleven cups to Indra, exactly as prescribed in the Vedas. Let s_1 denote the *citrā* sacrifice, o_1 denote cattle (as an outcome) and $a_1, \ldots, a_7$ denote, respectively, the offerings to Agni, to Soma, to Tvaṣṭṛ, to Sarasvatī, to Sarasvant, to Sinīvalī and to Indra. We do not need to talk about events in this sacrifice, so we can set $EVT = \emptyset$. Consider the model $\mathfrak{M} = \langle W, W_{\mathfrak{e}}, N^+, N^-, V \rangle$ s.t. $W = \{w_1, w_2, w_3\}$, $W_{\mathfrak{e}} = \{w_2, w_3\}$, $N^+(w_1) = N^-(w_1) = \{\{w_1, w_2\}, W\}$, $N^+(w_2) = N^-(w_2) = N^+(w_3) = N^-(w_3) = \emptyset$, $V(sub(a_j, s_1)) = W$ for $1 \leq j \leq 7$, $V(des(o_1)) = V(und(s_1)) = \{w_3\}$, $V(eap(a_k)) = V(amp(a_k)) = \{w_2, w_3\}$ for $1 \leq k \leq 6$, $V(amp(a_7)) = \{w_2, w_3\}$ and $V(eap(a_7)) = \{w_2\}$. Properties P1-P4 are clearly valid in $\mathfrak{M}$. It is easy to check that property P5 holds as well, given that all ritual actions $a_1, \ldots, a_7$ are performed exactly as prescribed by the Vedas only in situations in which they are performed as much as possible; more precisely, a_7 is the only action which is performed as much as possible in a situation, w_3, without being performed there exactly as prescribed by the Vedas—for instance, because one offers a nine-cup (rather than an eleven-cup) cake to Indra. The only relevant situation to check the validity of properties P6, P7 and P8 is w_1, since $N^+(w_2) = N^-(w_2) = N^+(w_3) = N^-(w_3) = \emptyset$. In the case of P6, the result follows from the fact that $\|\mathfrak{e} \rightarrow und(s_1)\|^{\mathfrak{M}} = \{w_1, w_3\} \notin N^+(w_1)$. P7 trivially holds since we have $EVT = \emptyset$. Finally, in the case of P8, given that $\|(\mathfrak{e} \wedge des(o_1)) \rightarrow und(s_1)\|^{\mathfrak{M}} = \|(\mathfrak{e} \wedge des(o_1)) \rightarrow (und(s_1) \rightarrow eap(a_k))\|^{\mathfrak{M}} = W$ for $1 \leq k \leq 6$ and $\|(\mathfrak{e} \wedge des(o_1)) \rightarrow (und(s_1) \rightarrow eap(a_7))\|^{\mathfrak{M}} = \{w_1, w_2\}$, then we have both $\|(\mathfrak{e} \wedge des(o_1)) \rightarrow und(s_1)\|^{\mathfrak{M}} \in N^-(w)$ and $\|(\mathfrak{e} \wedge des(o_1)) \rightarrow (und(s_1) \rightarrow eap(a_j))\|^{\mathfrak{M}} \in N^+(w)$ (for $1 \leq j \leq 7$), which is enough. Notice that in this model w_3 is a situation in which the duty to perform the *citrā* sacrifice is not observed, since one undertakes the sacrifice without performing all subsidiaries (in particular, the offering to Indra) exactly as prescribed. This is formally reflected by the fact that w_3 does not belong to all elements of the neighborhood sphere $N^+(w_1)$.

The next step is to prove the soundness of S_0, namely that every theorem of S_0 is valid in K_m. We use an induction on the length of derivations in S_0: first we show that all axiom-schemata of S_0 are valid in K_m and then that all rules of S_0 preserve validity in K_m. In some cases we can simply rely on well-known properties of standard neighborhood models, illustrated for instance in *Chellas* 1980.

[24] The *caru* is a cereal preparation offered in rituals. More details in *Mylius* 1995, s.v. and in *Einoo* 1985.

THEOREM 3.1 *For every $\phi \in \mathcal{L}$, if $\vdash_{S_0} \phi$ then $K_m \vDash \phi$.*

Proof Let $\mathfrak{M} = \langle W, W_{\mathfrak{e}}, N^+, N^-, V\rangle$ be an arbitrary model in K_m and w an arbitrary world (situation) in W. By definition of truth-conditions, $\mathfrak{M}, w \vDash \phi$ for any propositional tautology ϕ. This is enough for axiom (1). Suppose $\mathfrak{M}, w \vDash eap(a_i)$ for some $a_i \in ACT$; then, $w \in V(eap(a_i))$ and, by P5, we have $w \in V(amp(a_i))$, so $\mathfrak{M}, w \vDash amp(a_i)$, which is enough for axiom (3). Suppose $\mathfrak{M}, w \vDash fixed(s_i) \wedge sub(a_j, s_i)$, for some $s_i \in SAC$ and $a_j \in ACT$; then $\|\mathfrak{e} \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^+(w)$ and $w \in V(sub(a_j, s_i))$. By P6, $\|\mathfrak{e} \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}} \in N^+(w)$. This is enough for (4). One can similarly rely on P7 and P8 to show that every instance of (5) and (6) is true at w in $\mathfrak{M}$. Moreover, properties P1–P4 guarantee the validity of (8), (9), (11) and (12), exactly as in standard neighborhood models. Finally, rules (2), (7) and (10) preserve validity (rule (2) also preserves truth at a world) in every neighborhood model. ■

Theorem 3.1, together with the fact that K_m is a non-empty class of models (e.g. it includes the model for the *citrā* sacrifice used above) guarantees that S_0 is a *consistent* system.

The final step of our semantic characterization of S_0 consists in proving that this system is complete w.r.t. the class K_m, namely that every formula valid in K_m is also a theorem of S_0. We rely on the method of *canonical models*. We start by introducing some relevant terminology; what is here omitted for the sake of brevity can be found in *Chellas 1980*. So far we have been using a semantical notion of consistency for systems (a system X over $\mathcal{L}$ is consistent iff there exists a model for S); now, instead, we need to consider a syntactical notion of consistency: a system X over $\mathcal{L}$ is consistent iff there is at least some formula $\phi \in \mathcal{L}$ s.t. $\nvdash_{S_0} \phi$ (i.e. it is not the case that $\vdash_{S_0} \phi$). It can be shown that the two notions of consistency (semantic and syntactical) are equivalent. Furthermore, we need to define notions of *relative consistency*. A set of formulas $\Gamma \subset \mathcal{L}$ is consistent with S_0 iff there is no finite $\Gamma' \subseteq \Gamma$ s.t., if $\Gamma' = \{\phi_1, \ldots, \phi_n\}$, then $\vdash_{S_0} \neg(\phi_1 \wedge \cdots \wedge \phi_n)$. Finally, a set of formulas $\Gamma \subset \mathcal{L}$ is *maximally consistent* with S_0 iff (I) Γ is consistent with S_0 and (II) there is no $\Gamma' \subset \mathcal{L}$ s.t. $\Gamma \subset \Gamma'$ and Γ' is consistent with S_0.

Let the model $\mathfrak{M}_{S_0} = \langle W_{S_0}, W_{\mathfrak{e}S_0}, N^+_{S_0}, N^-_{S_0}, V_{S_0}\rangle$ be defined as follows:

- W_{S_0} is the set of all sets of formulas in $\mathcal{L}$ which are maximally consistent with S_0;
- for any $\phi \in \mathcal{L}$, $|\phi|_{S_0} = \{v \in W_{S_0} : \phi \in v\}$;
- $W_{\mathfrak{e}S_0} = |\mathfrak{e}|_{S_0}$;
- for every $w \in W_{S_0}$, $N^+(w) = \{X \subseteq \wp(W) :$ for some $\phi \in \mathcal{L}$, $|\phi|_{S_0} \subseteq X$ and $\mathcal{O}\phi \in w\}$;
- for every $w \in W_{S_0}$, $N^-(w) = \{X \subseteq \wp(W) :$ for some $\phi \in \mathcal{L}$, $|\phi|_{S_0} \subseteq X$ and $\mathcal{R}\phi \in w\}$;
- for any $at \in ATOM$, $V(at)_{S_0} = |at|_{S_0}$.

From the third and the sixth clause it follows that $V(\mathfrak{e})_{S_0} = W_{\mathfrak{e}S_0}$. According to its properties, $\mathfrak{M}_{S_0}$ can be called a *canonical model* for S_0.

THEOREM 3.2 *The model $\mathfrak{M}_{S_0}$ belongs to the class K_m.*

Proof The properties P1-P2 are trivially satisfied by $\mathfrak{M}_{S_0}$, by virtue of its definition. The property P3 is satisfied due to the fact that every $w \in W_{S_0}$ includes all instances of axiom (9). Indeed, suppose $X \in N^+(w)$ for some $X \in \wp(W)$, then there is $Y \in N(w)$ s.t. $Y \subseteq X$ and $Y = |\phi|_{S_0}$ for some $\phi \in \mathcal{L}$. This means that $\overline{X} \subseteq \overline{Y}$. According to the usual properties of canonical models, we know that $Y = |\phi|_{S_0} = \|\phi\|^{\mathfrak{M}_{S_0}}$, so $\mathfrak{M}_{S_0}, w \vDash \mathcal{O}\phi$ and, by axiom (9), $\mathfrak{M}_{S_0}, w \nvDash \mathcal{O}\neg\phi$; this entails $|\neg\phi|_{S_0} = \|\neg\phi\|^{\mathfrak{M}_{S_0}} = \overline{Y} \notin N^+(w)$. If it were the case that $\overline{X} \in N^+(w)$, then, by P1, it would also be the case that $\overline{Y} \in N^+(w)$ and this would represent a contradiction. The property P4 is satisfied due to the fact that every $w \in W$ includes all instances of axiom (12). Indeed, if it were the case that $\emptyset \in N^-(w)$, then, since for any $\phi \in \mathcal{L}$, $\emptyset = |\phi \wedge \neg\phi|_{S_0}$, we would have $\|\phi \wedge \neg\phi\|^{\mathfrak{M}_{S_0}} \in N^-(w)$, whence $\mathfrak{M}_{S_0}, w \vDash \mathcal{R}(\phi \wedge \neg\phi)$, which is impossible, given the validity of axiom (12). In the case of P5, let $w \in V_{S_0}(eap(a_i))$ for some $a_i \in ACT$; hence $\mathfrak{M}_{S_0}, w \vDash eap(a_i)$ and, by (3), we get $\mathfrak{M}_{S_0}, w \vDash amp(a_i)$, which entails $w \in V_{S_0}(amp(a_i))$. In the case of P6, let $w \in V_{S_0}(sub(a_j, s_i))$ for some $a_j \in ACT$ and $s_i \in SAC$; furthermore, let $\|\mathfrak{e} \rightarrow und(s_i)\|^{\mathfrak{M}_{S_0}} \in N^+(w)$, then $\mathfrak{M}_{S_0}, w \vDash sub(a_j, s_i) \wedge \mathcal{O}(\mathfrak{e} \rightarrow und(s_i))$; since every instance of (4) belongs to w, then we can infer $\|\mathfrak{e} \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}_{S_0}} \in N^+(w)$. The case of P7 and P8 can be dealt with in a similar way, relying on (5) and (6). ■

Taken together, Theorems 3.1 and 3.2 guarantee that S_0 is characterized by the class of models K_m.

Models for system S_0 can be divided into two classes depending on whether they satisfy the following additional property:

P9 for any $a_j \in ACT$ and $s_i \in SAC$, either $V(sub(a_j, s_i)) = W$ or $V(sub(a_j, s_i)) = \emptyset$.

This property means that the relation of subsidiarity among an action a_j and a sacrifice s_i either holds in all worlds/situations or in none. Models in which P9 is satisfied can be called *standard models* for S_0, those in which it is not satisfied *non-standard models* for S_0. The canonical model for S_0 used in Theorem 3.2 is actually a non-standard one, since there are at least two sets of formulas w and v which are maximally consistent with S_0 and s.t., for some $a_j \in ACT$ and $s_i \in SAC$, $sub(a_j, s_i) \in w$ while $sub(a_j, s_i) \notin v$. The model used earlier for the *citrā* sacrifice, instead, is a standard one.

Notice that the model for the *citrā* sacrifice also validates the schema $\mathcal{O}\phi \rightarrow \mathcal{R}\phi$, i.e. (13), since for all situations $v \in W$, we have $N^+(v) = N^-(v)$. Thus, that model can be also used to interpret the system S_1 described in Section 3.1, namely the system obtained by adding (13) to the axiomatic basis of S_0. This means that S_1 is a consistent system. In order to show the consistency of S_2, the other extension of S_0 mentioned in Section 3.1 and obtained by adding the schema $\mathcal{O}\phi \rightarrow \neg\mathcal{R}\phi$, i.e. (14), to the axiomatic basis of S_0, we provide a model based on the argument illustrating the duty to perform the *agnihotra* sacrifice.

Anyone who has studied the Vedas, is physically and economically able to perform a sacrifice (i.e. meets the eligibility conditions) has to perform the *agnihotra* sacrifice. Since it is a fixed sacrifice, its subsidiaries need to be performed as much as possible. Among the subsidiary actions to be performed there is an offering of milk obtained from a cow which has a male calf. Let s_1 stand for the *agnihotra* sacrifice and a_1 for the offering of the milk. Here we do not need to take into account events or desired outcomes, so we can set $EVT = OUT = \emptyset$. Take a model $\mathfrak{M} = \langle W, W_{\mathfrak{e}}, N^+, N^-, V\rangle$ where $W = \{w_1, w_2, w_3\}$, $W_{\mathfrak{e}} = \{w_2, w_3\}$, $N^+(w_1) = \{W\}$, $N^-(w_1) = N^+(w_2) = N^-(w_2) = N^+(w_3) = N^-(w_3) = \emptyset$, $V(sub(a_1, s_1)) = W$, $V(und(s_1)) = V(amp(a_1)) = \{w_2, w_3\}$ and $V(eap(a_1)) = \{w_2\}$. First, we show that this model belongs to the class K_m, so that it is a model for S_0. The validity of the properties P1–P4 is straightforward, given the definition of N^+ and N^-. The validity of P5 is also easy to check, since a_1 is the only ritual action mentioned and $V(eap(a_1)) \subseteq V(amp(a_1))$. More precisely, $eap(a_1)$ holds only in the situation w_2, whereas $amp(a_1)$ holds in both w_2 and w_3 (for instance, because in w_3 the milk is obtained from a cow which has only a female calf). The validity of P7 and P8 is trivial since $EVT = OUT = \emptyset$. Thus, it only remains to check the validity of P6: given that

$\|\mathfrak{e} \rightarrow und(s_1)\|^{\mathfrak{M}} = \|\mathfrak{e} \rightarrow (und(s_1) \rightarrow amp(a_1))\|^{\mathfrak{M}} = W$, then $\|\mathfrak{e} \rightarrow und(s_1)\|^{\mathfrak{M}}, \|\mathfrak{e} \rightarrow (und(s_1) \rightarrow amp(a_1))\|^{\mathfrak{M}} \in N^+(w)$ and this is enough. Finally, we have to show that the schema (14), i.e. $\mathcal{O}\phi \rightarrow \neg\mathcal{R}\phi$, is valid in our model. The only relevant situation to consider is w_1 (given that $N^+(w_2) = N^-(w_2) = N^+(w_3) = N^-(w_3) = \emptyset$) and the result holds since $N^+(w_1) \cap N^-(w_1) = \emptyset$. Thus, this is a model for S_2 and the latter is a consistent system, as claimed in Section 3.1. Notice that in the model for the *agnihotra* sacrifice all strong and weak Vedic duties considered are observed in the three situations w_1, w_2 and w_3; indeed, the three situations belong to every element of their neighborhood spheres (in most cases, trivially, since all neighborhood spheres except $N^+(w_1)$ are empty).

3.3. *Variation: dyadic deontic operators*

Vedic injunctions concerning the performance of ritual actions can be formalized also in terms of *dyadic deontic operators*, namely in a language in which $\mathcal{O}$ and $\mathcal{R}$ always take two formulas as arguments rather than one. Formulas whose main operator is either dyadic $\mathcal{O}$ or dyadic $\mathcal{R}$ will be here represented as $\mathcal{O}_\theta\phi$ and $\mathcal{R}_\theta\phi$; another common notation is $\mathcal{O}(\phi/\theta)$ and $\mathcal{R}(\phi/\theta)$. Notice that in both cases ϕ represents the main argument and θ the antecedent or *triggering condition* (despite being positioned after ϕ in the second notation; that is due to alternative natural language translations of these formulas, such as 'it should be the case that condition θ leads one to bring about ϕ' versus 'it should be the case that one brings about ϕ under condition θ').[25]

In the present context, dyadic operators can be useful to express the fact that eligibility is always a triggering condition for the performance of a sacrifice. Moreover, according to Mīmāṃsā authors, each Vedic injunction addresses a person insofar as he or she is identified by a particular desire. In this perspective, desire is an essential element of the 'eligibility' (*adhikāra*, see on this point *Freschi 2007*) so much that, when no desire is explicitly mentioned in connection with an injunction, these authors explain that a desire for happiness can be assumed to fill this necessary slot (such procedure is called *viśvajinnyāya* and is discussed in *Exegetic Aphorisms* 4.3.15–16). A similar argument actually leads to two possible solutions to formalize the Mīmāṃsā theory. One solution consists in taking the desire for happiness as redundant, following the idea that this desire is shared by all human beings. The other solution consists in making explicit reference to the desire for happiness when no other specific desire is mentioned. In Section 3.1 we opted for the first solution; here, for a comparison, we will opt for the second solution and show how the definition of fixed and occasional sacrifices can be changed by adding 'happiness' as a special desired outcome. Before doing that, however, we would like to mention another issue concerning the move from monadic to dyadic deontic operators: the treatment of deontic dilemmas.

Dyadic deontic operators are exploited in *Ciabattoni et al. 2015* to formalize an argument discussed by Mīmāṃsā authors, which can be called the *śyena dilemma*. Such dilemma pertains to the performance of the *śyena* sacrifice, a malefic sacrifice meant for harming one's enemy. Mīmāṃsā authors all agree that it should not be performed because it violates the prohibition to harm living beings. *Ciabattoni et al. 2015* argue that the seeming conflict between the prohibition to harm any living being and the injunction to perform the *śyena* under the desire to harm an enemy can be formally solved through a dyadic operator. Indeed, it is possible to build a formal system, like **bMDL**, in which the dilemma is

[25] For an introduction to the use of dyadic operators in deontic logic, see *Hilpinen and McNamara 2013*. We adhere to the notation in *Åqvist 1987*.

represented by a conjunction of formulas $\mathcal{O}_{\top}\neg\phi \wedge \mathcal{O}_{\psi}\phi$, where ϕ can be read 'you harm some living being' (which is a necessary consequence of the performance of the *śyena*), ψ 'you desire to harm your enemy' and $\top$ represents any tautology. The point is that from the conjunction $\mathcal{O}_{\top}\neg\phi \wedge \mathcal{O}_{\psi}\phi$ one cannot infer, in **bMDL**, both $\mathcal{O}_{\psi}\neg\phi$ and $\mathcal{O}_{\psi}\phi$, which would lead to the conclusion that under the same condition ψ ('you desire to harm your enemy') you should both harm and not harm other living beings.

Within the conceptual framework developed in this article, which allows one to distinguish between *strong* and *weak* Vedic duties, we can provide an alternative analysis of the *śyena* dilemma. Indeed, since the *śyena* sacrifice has to be performed when a desire for something different from happiness arises (harming an enemy), then it can be classified as an elective sacrifice. This means, according to our analysis in Section 2.1.4, that the injunction to perform it expresses a *weak* duty or recommendation. On the other hand, the injunction to avoid harming any living being represents a generic prohibition. Thus, in the present conceptual framework the dilemma would no longer involve two obligations but a recommendation and a prohibition: one could say that the *śyena* sacrifice is something whose performance is *recommended* in situations that are not deontically ideal, since they are situations in which one desires something that is, in general, *prohibited* (harming some living being and, more specifically, an enemy).[26]

The possible applications of dyadic operators discussed so far, including the treatment of deontic dilemmas, suggest a generalization of the formal apparatus of Section 3.1. We briefly illustrate how this generalization can be achieved. Let $\mathcal{L}d$ be the language obtained by replacing monadic $\mathcal{O}$ and $\mathcal{R}$ in $\mathcal{L}$ with their dyadic analogue. The set of wffs of $\mathcal{L}d$ is then the smallest set satisfying the following properties:

- at is a wff, for any $at \in ATOM$;
- if ϕ is a wff, then so is $\neg(\phi)$;
- if ϕ and ψ are wffs, then so are $(\phi) \rightarrow (\psi)$, $\mathcal{O}_{\psi}(\phi)$ and $\mathcal{R}_{\psi}(\phi)$.

Parentheses can be eliminated when there is no risk of ambiguity, as usual. The meaning of $\mathcal{O}_{\psi}(\phi)$ and $\mathcal{R}_{\psi}(\phi)$ is, respectively, 'in all situations in which strong Vedic duties are observed, ϕ is the case under condition ψ' and 'in all situations in which weak Vedic duties are observed, ϕ is the case under condition ψ'. Our simplified reading, then, turns out to be: 'according to the Vedas, ϕ is obligatory given condition ψ' and 'according to the Vedas, ϕ is recommended given condition ψ'.

Since here we want to formalize injunctions by making explicit reference to the desire for happiness when no other specific desire is mentioned, then we can assume that the set of outcomes OUT (which is inherited, by definition of $\mathcal{L}d$, from the language $\mathcal{L}$ of Section 3.1) includes a special individual constant o^* denoting *happiness* and we can provide the following definitions of *fixed*, *occasional* and *elective* sacrifices:

- $fixed(s_i) =_{def} \mathcal{O}_{\mathfrak{e} \wedge des(o^*)}(und(s_i))$;
- $occasional(s_i)/e_n =_{def} \mathcal{O}_{\mathfrak{e} \wedge tpl(e_n) \wedge des(o^*)}(und(s_i))$;
- $elective(s_i)/o_n =_{def} \mathcal{R}_{\mathfrak{e} \wedge des(o_n)}(und(s_i))$.

The first definition says that s_i denotes a fixed sacrifice iff in all situations in which strong Vedic duties are observed, eligibility is met and happiness is desired, s_i is undertaken. The second definition says that s_i denotes an occasional sacrifice iff in all situations in which strong Vedic duties are observed, eligibility is met, the relevant occasion takes place and

[26] Prohibitions cannot be simply defined as negative obligations in the Mīmāṃsā theory, since prohibitions are defined insofar as their transgression leads to sanctions, whereas obligations, even negative obligations, lead to a result. For a systematic overview of deontic commands in Mīmāṃsā, see *Freschi and Pascucci 2019*.

happiness is desired, s_i is undertaken. The third definition says that s_i denotes an elective sacrifice iff in all situations in which weak Vedic duties are observed, eligibility is met and the relevant outcome is desired, s_i is undertaken. These definitions differ from the original ones in Section 3.1 due to the presence of o^* (happiness) when no other outcome is desired and to the fact that the dyadic notation replaces the main material implication in the scope of $\mathcal{O}$ and $\mathcal{R}$.

We introduce now a formal system called S_0d, which is the dyadic analogue of S_0. We can borrow the principles (1), (2) and (3) directly from S_0. The principles (4), (5) and (6), instead, need to be modified by replacing the main material implication in the scope of $\mathcal{O}$ with the dyadic notation and by taking into account o^* when fixed and occasional sacrifices are mentioned. For the sake of simplicity, we will refer to the modified principles with the same numbers used for the original ones. For instance, the schema (5) becomes: $(occasional(s_i)/e_n \wedge sub(a_j, s_i)) \rightarrow \mathcal{O}_{\mathfrak{e} \wedge tpl(e_n) \wedge des(o^*)}(und(s_i) \rightarrow amp(a_j))$.

In order to axiomatize the properties of the dyadic operator $\mathcal{O}$, we choose to rely again on the preliminary system **bMDL** developed in *Ciabattoni et al. 2015*. In the Appendix to this article it is shown that we can extract from **bMDL** the *dyadic versions* of the principles (7), (8) and (9),[27] together with the following rule:

$$\frac{\theta \equiv \xi}{\mathcal{O}_\theta \phi \equiv \mathcal{O}_\xi \phi}. \tag{15}$$

Similarly, as far as the principles for dyadic $\mathcal{R}$ are concerned, we take the *dyadic versions* of the principles (10), (11) and (12),[28] together with the rule below:

$$\frac{\theta \equiv \xi}{\mathcal{R}_\theta \phi \equiv \mathcal{R}_\xi \phi}. \tag{16}$$

The axiomatic basis of S_0d is finally defined by the list of (modified) principles (1)–(12), together with (15) and (16).

In the rest of this section we sketch how the semantic characterization of S_0 can be modified in order to get a semantic characterization of S_0d. Models are now to be defined as structures of kind $\mathfrak{M} = \langle W, W_\mathfrak{e}, \{N^+_\theta : \theta \in \mathcal{L}d\}, \{N^-_\theta : \theta \in \mathcal{L}d\}, V\rangle$. Thus, for every situation $w \in W$ and every formula $\theta \in \mathcal{L}d$ we have now a pair of neighborhood spheres, $N^+_\theta(w)$ and $N^-_\theta(w)$. Truth conditions are as in models for S_0, except those concerning formulas whose main operator is dyadic $\mathcal{O}$ or dyadic $\mathcal{R}$:

- $\mathfrak{M}, w \models \mathcal{O}_\theta \phi$ iff $\|\phi\|^{\mathfrak{M}} = \{v \in W : \mathfrak{M}, v \models \phi\} \in N^+_\theta(w)$;
- $\mathfrak{M}, w \models \mathcal{R}_\theta \phi$ iff $\|\phi\|^{\mathfrak{M}} = \{v \in W : \mathfrak{M}, v \models \phi\} \in N^-_\theta(w)$.

Let $K_m d$ be the class of models satisfying a modified version of the properties P1-P8 of models in K_m (see Section 3.2), obtained in the following way:

- in P1 and P3 replace any occurrence of $N^+(w)$ with $N^+_\theta(w)$, for an arbitrary $\theta \in \mathcal{L}d$;
- in P2 and P4 replace any occurrence of $N^-(w)$ with $N^-_\theta(w)$, for an arbitrary $\theta \in \mathcal{L}d$;

[27] Namely, $\frac{\phi \equiv \psi}{\mathcal{O}_\theta \phi \equiv \mathcal{O}_\theta \psi}$, $\mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta \phi \wedge \mathcal{O}_\theta \psi)$ and $\neg(\mathcal{O}_\theta \phi \wedge \mathcal{O}_\theta \neg\phi)$; we will keep the original numbers for reference.

[28] Namely, $\frac{\phi \equiv \psi}{\mathcal{R}_\theta \phi \equiv \mathcal{R}_\theta \psi}$, $\mathcal{R}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{R}_\theta \phi \wedge \mathcal{R}_\theta \psi)$ and $\neg\mathcal{R}_\theta(\phi \wedge \neg\phi)$; we will keep the original numbers for reference.

- in P6 replace $\|\mathfrak{e} \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^+(w)$ with $\|und(s_i)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge des(o^*)}(w)$ and $\|\mathfrak{e} \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}} \in N^+(w)$ with $\|und(s_i) \rightarrow amp(a_j)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge des(o^*)}(w)$;
- in P7 replace $\|(\mathfrak{e} \wedge tpl(e_n)) \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^+(w)$ with $\|und(s_i)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge tpl(e_n)\wedge des(o^*)}(w)$ and $\|(\mathfrak{e} \wedge tpl(e_n)) \rightarrow (und(s_i) \rightarrow amp(a_j))\|^{\mathfrak{M}} \in N^+(w)$ with $\|und(s_i) \rightarrow amp(a_j)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge tpl(e_n)\wedge des(o^*)}(w)$;
- in P8 replace $\|(\mathfrak{e} \wedge des(o_n)) \rightarrow und(s_i)\|^{\mathfrak{M}} \in N^-(w)$ with $\|und(s_i)\|^{\mathfrak{M}} \in N^-_{\mathfrak{e}\wedge des(o_n)}(w)$ and $\|(\mathfrak{e} \wedge des(o_n)) \rightarrow (und(s_i) \rightarrow eap(a_j))\|^{\mathfrak{M}} \in N^+(w)$ with $\|und(s_i) \rightarrow amp(a_j)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge des(o_n)}(w)$.

Furthermore, let models in $K_m d$ satisfy the following additional property:

P10 if $\vdash_{S_0 d} \theta \equiv \xi$, then $N^+_\theta(w) = N^+_\xi(w)$ and $N^-_\theta(w) = N^-_\xi(w)$.

It can be easily shown that there is at least one model in the class $K_m d$. We provide here a model based on the argument used to enjoin the performance of the *rājanīṣṭi* sacrifice. A person who desires to improve his sight and fulfills the other relevant criteria for eligibility (e.g. he or she has studied the Vedas, is physically able and is wealthy enough) is the addressee of the injunction to perform the *rājanīṣṭi*. Since this is an elective sacrifice, one can reach its result only by performing all its auxiliaries exactly as prescribed; in this case, the offering of two eight-cup cakes for Agni bhrājasvant and the offering of a *caru* for Sūrya. Let s_1 stand for the *rājanīṣṭi* sacrifice, a_1 for the offering of two eight-cup cakes for Agni bhrājasvant, a_2 for the offering of a *caru* for Sūrya and o_1 for the outcome of improving one's sight. Here we do not need to take into account events so we can set $EVT = \emptyset$. Take a model $\mathfrak{M} = \langle W, W_\mathfrak{e}, \{N^+_\theta : \theta \in \mathcal{L}d\}, \{N^-_\theta : \theta \in \mathcal{L}d\}, V\rangle$ where $W = \{w_1, w_2, w_3\}$, $W_\mathfrak{e} = W$, $N^+_{\mathfrak{e}\wedge des(o_1)}(w_1) = \{\{w_1, w_3\}, W\}$, $N^-_{\mathfrak{e}\wedge des(o_1)}(w_1) = \{\{w_1, w_2\}, W\}$, $N^+_\xi(w_1) = N^-_\xi(w_1) = \emptyset$ for any $\xi \in \mathcal{L}d$ s.t. $\xi \neq (\mathfrak{e} \wedge des(o_1))$, $N^+_\theta(w_2) = N^-_\theta(w_2) = N^+_\theta(w_3) = N^-_\theta(w_3) = \emptyset$ for any $\theta \in \mathcal{L}d$, $V(sub(a_1, s_1)) = V(sub(a_2, s_1)) = V(des(o_1)) = W$, $V(und(s_1)) = V(amp(a_1)) = V(amp(a_2)) = V(eap(a_2)) = \{w_1, w_2\}$ and $V(eap(a_1)) = \{w_1\}$. The validity of the properties P1-P4 follows from the definition of N^+_θ and N^-_θ, for any $\theta \in \mathcal{L}d$. P5 holds as well, since a_1 and a_2 are the only two ritual actions mentioned and $V(eap(a_2)) = V(amp(a_2))$, while $V(eap(a_1)) \subset V(amp(a_1))$. More precisely, $eap(a_1)$ holds only in the situation w_1, whereas $amp(a_1)$ holds in the situations w_1 and w_2 (for instance, because in w_2 one offers two five-cup cakes, rather than two eight-cup cakes, for Agni bhrājasvant). The validity of P6 and P7 is trivial since $N^+_{\mathfrak{e}\wedge des(o^*)}(w) = \emptyset$ for any $w \in W$ and $EVT = \emptyset$. It only remains to show the validity of P8 and the only situation to check is w_1: given that $\|und(s_1)\|^{\mathfrak{M}} = \{w_1, w_2\}$, $\|und(s_1) \rightarrow eap(a_1)\|^{\mathfrak{M}} = \{w_1, w_3\}$ and $\|und(s_1) \rightarrow eap(a_2)\|^{\mathfrak{M}} = W$, then $\|und(s_1)\|^{\mathfrak{M}} \in N^-_{\mathfrak{e}\wedge des(o_1)}(w_1)$ and $\|und(s_1) \rightarrow eap(a_1))\|^{\mathfrak{M}}, \|und(s_1) \rightarrow eap(a_2)\|^{\mathfrak{M}} \in N^+_{\mathfrak{e}\wedge des(o_1)}(w_1)$ and this is enough. Notice that in the model for the *rājanīṣṭi* sacrifice w_1 is a situation in which all strong and weak Vedic duties mentioned are observed; indeed, it belongs to all elements of all its neighborhood spheres. The same applies, trivially, also to w_2 and w_3.

The soundness of $S_0 d$ w.r.t. the class $K_m d$ can be easily checked; the only relevant remark is that P10 guarantees that (15) and (16) preserve validity in all models of $K_m d$. As far as completeness is concerned, a canonical model $\mathfrak{M}_{S_0 d}$ for $S_0 d$ can be defined in analogy with the model $\mathfrak{M}_{S_0}$ of Section 3.2, except for the properties concerning neighborhood spheres:

- for every $w \in W_{S_0 d}$, $N^+_\theta(w) = \{X \subseteq \wp(W) :$ for some $\phi \in \mathcal{L}d$, $|\phi|_{S_0 d} \subseteq X$ and $\mathcal{O}_\theta \phi \in w\}$;

– for every $w \in W_{S_0d}$, $N^-_\theta(w) = \{X \subseteq \wp(W) :$ for some $\phi \in \mathcal{L}d$, $|\phi|_{S_0d} \subseteq X$ and $\mathcal{R}_\theta\phi \in w\}$;

It can be finally verified that $\mathfrak{M}_{S_0d}$ satisfies the modified version of P1-P8 and P10, so that it belongs to the class K_md. Here we just consider the case of P10. Assume that $\vdash_{S_0d} \theta \equiv \xi$; then, by properties of canonical models, for every world $w \in W_{S_0d}$, we have $\mathfrak{M}_{S_0d}, w \vDash \theta \equiv \xi$; from this, by (15) and (16) one gets that for every formula $\phi \in \mathcal{L}d$, $\mathfrak{M}_{S_0d}, w \vDash (\mathcal{O}_\theta\phi \equiv \mathcal{O}_\xi\phi) \wedge (\mathcal{R}_\theta\phi \equiv \mathcal{R}_\xi\phi)$. Thus, (I) $\|\phi\|^{\mathfrak{M}^{S_0d}} \in N^+_\theta$ iff $\|\phi\|^{\mathfrak{M}^{S_0d}} \in N^+_\xi$ and (II) $\|\phi\|^{\mathfrak{M}^{S_0d}} \in N^-_\theta$ iff $\|\phi\|^{\mathfrak{M}^{S_0d}} \in N^-_\xi$. From this we get the intended result, namely that (I) $N^+_\theta(w) = N^+_\xi(w)$ and (II) $N^-_\theta(w) = N^-_\xi(w)$.

We conclude this section with a remark on useful variations of S_0d. We argued that one of the reasons to move from monadic deontic operators to dyadic ones is the possibility of treating deontic dilemmas. For instance, as illustrated in the analysis of the *śyena* dilemma in *Ciabattoni et al.* 2015, one can block problematic inferences from a generic triggering condition to a more specific one; however, if the only principles relating triggering conditions are those formalized by (15) and (16), there is some risk that too many inferences are blocked. For instance, it seems plausible to assume that sometimes the addition of a triggering condition preserves the injunction to perform an action. Consider the case of the *rājanīṣṭi* sacrifice discussed above. If it is the case that a person should perform the *rājanīṣṭi* when he or she is eligible and desires to improve his or her sight, then it seems also plausible that that person should perform the *rājanīṣṭi* when, in addition to the two conditions mentioned, he or she also has a generic desire for happiness. However, in the system S_0d over the language $\mathcal{L}d$ it is not possible to make this inference, since the formula $\mathcal{R}_{\mathfrak{e}\wedge des(o_1)}(und(s_1))$ does not entail the formula $\mathcal{R}_{\mathfrak{e}\wedge des(o_1)\wedge des(o^*)}(und(s_1))$, where s_1 and o_1 are defined as in the *rājanīṣṭi* example and o^* denotes happiness. Similar issues arise with dyadic $\mathcal{O}$ (already in the system **bMDL**, whence we extracted our principles for this operator). Thus, there is some need to add further principles relating triggering conditions in systems built over $\mathcal{L}d$. Let S_0d^* be the system obtained by extending the axiomatic basis of S_0d with the two principles below:

$$\mathcal{O}_\psi\phi \to \mathcal{O}_{\psi\wedge des(o^*)}\phi, \tag{17}$$

$$\mathcal{R}_\psi\phi \to \mathcal{R}_{\psi\wedge des(o^*)}\phi. \tag{18}$$

The meaning of these two principles is intuitive: if it is a (strong or weak) Vedic duty to perform ϕ under condition ψ, then it is also a (strong or weak) Vedic duty to perform ϕ under the condition ψ and the desire for happiness. These two principles guarantee, at least, that strong and weak duties are always preserved under the addition of the desire for happiness. On the other hand, if one wants to claim that the desire for happiness not only preserves duties, but makes no difference at all with respect to duties, then she can add to the axiomatic basis of S_0d^* also the following principles:

$$\mathcal{O}_{\psi\wedge des(o^*)}\phi \to \mathcal{O}_\psi\phi, \tag{19}$$

$$\mathcal{R}_{\psi\wedge des(o^*)}\phi \to \mathcal{R}_\psi\phi. \tag{20}$$

The result is a system that we can call S_0d^{**}. This system brings one back to the idea that reference to the desire for happiness is redundant and provides also a formal justification of this position. Indeed, even if o^* is mentioned in the definition of fixed and occasional sacrifices, in S_0d^{**} any formula in which o^* occurs as a triggering condition (having the shape

$\mathcal{O}_{\psi \wedge des(o^*)}\phi$ or $\mathcal{R}_{\psi \wedge des(o^*)}\phi$) is logically equivalent to the formula obtained by removing o^* from the set of triggering conditions (having the shape $\mathcal{O}_{\psi}\phi$ or $\mathcal{R}_{\psi}\phi$).

4. Maṇḍana Miśra's interpretation of Vedic duties

According to our analysis in Section 2, the deontic theory of Vedic sacrifices developed in Common Mīmāṃsā is characterized by the following features:

1. it seems to entail *several kinds of Vedic duties*, which can be classified according to the fundamental distinction between *strong* duties (obligations) and *weak* duties (recommendations);
2. it seems to entail *three irreducible classes of ritual actions*, namely fixed, occasional and elective ones;
3. it seems to entail a role for *both* (positive) *results* and (negative) *sanctions*, though without specifying it clearly.

We have seen how these distinctions are logically connected: Certain principles, according to Mīmāṃsā, allow us to infer deontic features of an action from, for example, the fact that it is a subsidiary of an elective sacrifice. We have not, however, seen any attempt from within Common Mīmāṃsā to 'rationalize' this system of principles, by which we mean both giving a rational motivation for the principles themselves (e.g. what is the reason why subsidiaries of fixed sacrifices only need to be performed to the best of one's ability, in contrast to the subsidiaries of elective sacrifices, which need to be performed exactly as the texts prescribe?) as well as identifying and eliciting the basic concepts on which the system is based.

The eighth-century Mīmāṃsā author Maṇḍana Miśra offered just such a rationalization in his *Analysis of Injunctions* (*Vidhiviveka*). He argued that what we called the Common Mīmāṃsā system could be streamlined by interpreting the meaning of injunctions (*vidhiliṅpratyayārtha*) in a uniform way across all cases. The reduction that he effected has a twofold sense:

– the reduction of multiple concepts of injunction, and classes of actions that can be enjoined, to a single concept of injunction and a homogeneous class of actions. This deals with the issues enunciated above as point 1 and point 2;
– the reduction of deontic concepts to non-deontic concepts, and in particular, the reduction of the meaning of injunctions to a form of instrumentality, namely to the idea that enjoined actions are instruments to get a desired result. In this way, he attempted to rethink the role of results and addressed the issue raised above as point 3.

In other words, Maṇḍana reduced the various types of sacrifices to a single case, insofar as he argued that all sacrifices are based on a desire. Furthermore, he reinterpreted injunctions in non-deontic terms as just stating that the prescribed action is an instrument to realize a desired output. In this way, Maṇḍana replaced multiple notions of duty and injunctions with one by reducing the notion of duty to that of instrumentality towards a desired result, and the notion of injunction to that of a statement that enunciates an action's instrumental relation to a desired result.

The essential point for Maṇḍana is that all three classes of action share the property of leading to a certain result. We can articulate his reduction in four steps:

Step one: All sacrifices have a result. Maṇḍana postulates that fixed sacrifices lead to fixed results (i.e. stable, permanent results, such as happiness), and elective sacrifices lead to non-fixed results (i.e. things that happen once, such as the acquisition of cattle).

Step two: Occasional sacrifices are deontically identical to fixed sacrifices. This is implicit already in early Mīmāṃsā, so Maṇḍana did not have to argue at length for this position. The basic idea is that both fixed and occasional sacrifices have an occasion. The occasion for the former recurs on a regular basis, whereas the occasion for the latter does not.

One does not need to be concerned about the fact that, for example, the occasion of the *agnihotra* sacrifice takes effect every day, while the occasion of the *jyotiṣṭoma* sacrifice takes effect every year and the occasion of the *jāteṣṭi* sacrifice takes effect only a couple of times in one's life. This position is supported by the following observation:

> An occasion is that of which the presence makes the performance of an action necessary.[29] How is it so? The principal cause (in this case, the occasion) is the one which, being present, [the result] arises, all other [causes] are secondary (similarly, when the occasion is present, the sacrifice needs to be performed).[30]

An occasion is not brought about by the agent's effort (*anupādeya*), in the sense that its occurrence is completely independent of the agent's will and desire. Maṇḍana considered 'living' (*jīvana*) the occasion of both the fixed and occasional rituals. In both cases 'living' is qualified by a particular *time*.[31] The difference is that the time of the fixed rituals is simply daily (i.e. every evening, every morning, etc.), whereas the time of the so-called occasional rituals is either less frequent (e.g. every springtime) or less predictable (e.g. upon the birth of a son).

An action whose necessary performance is 'triggered' by an occasion does not need to be performed with all of its subsidiaries present. Rather, the action to be performed is to be provided with only those subsidiaries that it is feasible for the agent to include.[32] Generally an injunction conveys that one who performs the enjoined act, when the occasion is present, will accomplish the end he desires. Maṇḍana maintained that he will *always* accomplish this end *so long as he performs the enjoined action in some way or another* (*yathākathaṃcid*).[33] When the occasion is 'living', we know, additionally, that in order for the action to be accomplished on the given occasion (i.e. throughout one's life), some allowances have to be made for times of adversity, hardship, etc., and therefore the 'good enough' principle is invoked (see below, step four).

Step three: Fixed and occasional sacrifices have a result that is always desired. Maṇḍana argued that it is *necessary* in Mīmāṃsā to connect even the fixed and occasional sacrifices with a result, namely happiness. The result is also necessary to connect the principal act (e.g. the performance of the full- and new-moon sacrifices) with subsidiary acts (e.g. the performance of the preliminary sacrifices), since the latter assist in bringing about the result of the former, which therefore in their case fulfils the role of the desired end. When a result is not specifically mentioned it has to be postulated.

Step four: Elective sacrifices have a result that is sometimes desired. The result of these sacrifices is something that one happens to desire in non-ordinary circumstances, for instance rain in case of drought.

29 *Analysis of Injunctions* in *Gosvami 1978*, p. 256: *yatra jāte karmmaṇo 'vaśyakarttavyatā tan nimittam.*

30 *Analysis of Injunctions* p. 256: *katham? yasmin sati bhavaty eva sa hetur mukhyaḥ, itaras tu bhaktyā.*

31 *Analysis of Injunctions* p. 245: *jīvanāder nimittasya sāmyāt kālo viśeṣakaḥ — nimittārthas tatra jāte karmmaṇo 'vaśyakāryyatā.*

32 *Analysis of Injunctions* p. 257: *avaśyakarttavyatā ca karmmaṇaḥ pratītā nopasaṃhāryyanikhilāṅgasya kalpate, na kadācid ahitopāyasyety ato yathāśaktyaṅgasamavetaṃ sadā samabhilaṣitopāyaṃ karmmeti gamyate.*

33 *Analysis of Injunctions* p. 257: *nimittatā ca karttavyatāyāḥ vidhirūpāyāḥ. tasmin sati tatkurvato yathākathaṃcid sadā samīhitam enaḥkṣayaṃ sādhayati.*

Thus, summing up, a result which one constantly desires motivates one to undertake a fixed sacrifice throughout one's life and an occasional sacrifice whenever the relevant occasion occurs; a result which one contingently desires motivates one to undertake the corresponding elective sacrifice whenever one desires its result. Maṇḍana's proposal seems to entail that all injunctions actually express recommendations: injunctions can be simply taken as recipes *useful* to reach a desired goal. Notice that, according to our analysis, recommendation is the kind of deontic property that can be ascribed to elective sacrifices and, according to Maṇḍana's perspective, all sacrifices, like the elective sacrifices of Common Mīmāṃsā, are performed only because one desires their result. Thus, Maṇḍana's argument first eliminates any reference to obligations and then suggests an alternative reading for recommendations: something is recommended iff it is an instrument to achieve a desired goal. It can be argued that his attempt to provide a completely uniform account of all sacrifices could undermine the distinction between subsidiary actions that have to be performed exactly as prescribed and subsidiary actions that have to be performed according to one's capacity (the 'good enough' principle). Maṇḍana was very concerned, however, to retain the interpretive and ritual conclusions of Common Mīmāṃsā, while putting them on a new theoretical foundation. Therefore he was at pains to distinguish subsidiaries that have to be performed exactly as enjoined and those that can be performed as much as possible within his new framework, and to this end, he invoked the principle that the Veda cannot enjoin the performance of something that is physically impossible, and therefore allowances must be made for cases of exigency.

From a formal point of view, Maṇḍana's argument could be taken into account by replacing the definitions of fixed and occasional sacrifices in Section 3.3 with those below (where o^*, as usual, represents happiness):

- $\mathit{fixed}(s_i) =_{def} \mathcal{R}((\mathfrak{e} \wedge des(o^*)) \rightarrow und(s_i))$;
- $\mathit{occasional}(s_i)/e_n =_{def} \mathcal{R}((\mathfrak{e} \wedge tpl(e_n) \wedge des(o^*)) \rightarrow und(s_i))$;

The main differences with respect to the old definitions are:

- the use of $\mathcal{R}$ rather than $\mathcal{O}$ as the main operator in the *definiens* of fixed and occasional sacrifices;
- reference to the desire for happiness as a triggering condition in the *definiens* of fixed sacrifices and occasional sacrifices.[34]

Alternatively, in a dyadic setting, one can replace the definitions of fixed and occasional sacrifices in Section 3.3 (which already take into account the role of the desire for happiness) with the two below:

- $\mathit{fixed}(s_i) =_{def} \mathcal{R}_{\mathfrak{e} \wedge des(o^*)}(und(s_i))$;
- $\mathit{occasional}(s_i)/e_n =_{def} \mathcal{R}_{\mathfrak{e} \wedge tpl(e_n) \wedge des(o^*)}(und(s_i))$.

Both solutions allow one to capture the idea that, according to Maṇḍana, Vedic injunctions to perform sacrifices have to be always taken as recipes rather than orders. Thus, they have the deontic force of a recommendation, rather than an obligation.

5. Conclusion

The article shows some of the results of applying conceptual analysis and formal logic to the understanding of a premodern philosophical tradition and how one gains through this

[34] The idea of making explicit reference to a desire in each sacrifice has been followed already in the formalism of Section 3.3. Maṇḍana strengthens the Common Mīmāṃsā principle that desires are always present as triggering conditions for the performance of sacrifices, by saying that one undertakes sacrifices *only* in order to fulfill a desire.

exercise both clarity in one's understanding of the tradition and new insights, in general, regarding logic and deontic reasoning. We saw how one can make sense of the Mīmāṃsā distinction between the duties to perform fixed, occasional and elective sacrifices by focusing on the interplay between certain deontic concepts, such as those of strong duty and weak duty (obligation and recommendation), the role of primary and subsidiary ritual actions, the eligibility criterion and the 'good enough' principle.

We proposed formalisms that can be used to represent arguments discussed by Mīmāṃsā authors in a rather easy way and introduced logical systems capturing different properties of weak duties and strong duties. These ideas can be further developed to reach a more refined analysis of the deontic theory outlined by Mīmāṃsā. Furthermore, as a side aspect, we illustrated some connection between the formal systems employed here and a system used in earlier works on Mīmāṃsā, **bMDL** (see also the Appendix, which is dedicated to the proof of how one can extract axioms and rules for the operator $\mathcal{O}$ from the latter system).

Finally, the analysis elaborated in this article gave us a philosophical perspective onto the intervention of Maṇḍana on the Common Mīmāṃsā system, which turned out to be a simplification of the earlier description of ritual actions and a reduction of various deontic notions of duty to a non-deontic notion of instrumentality. Future research will aim at developing a detailed formal analysis of Maṇḍana's intervention.

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Appendix. Properties of the operator $\mathcal{O}$

From Mīmāṃsā texts to axioms

In *Ciabattoni et al.* (*2015*), the authors introduce a system called **bMDL** (*basic Mīmāṃsā deontic logic*), which is obtained by adding to (any axiomatization of) the alethic system **S4** the following axioms, where $\Box$ is the alethic operator for necessity:[35]

$$(\Box(\phi \to \psi) \land \mathcal{O}_\theta \phi) \to \mathcal{O}_\theta \psi, \tag{A1}$$

$$\Box(\psi \to \neg\phi) \to \neg(\mathcal{O}_\theta \phi \land \mathcal{O}_\theta \psi), \tag{A2}$$

$$(\Box((\psi \to \theta) \land (\theta \to \psi)) \land \mathcal{O}_\psi \phi)) \to \mathcal{O}_\theta \phi. \tag{A3}$$

The above axioms arise by formalizing some of the principles (called *nyāya*s) lied down by Mīmāṃsā authors to interpret the Vedas independently of any authorial intention. *Nyāya*s are metarules introduced implicitly or explicitly by Mīmāṃsā authors in order to bring order in their exegesis of Vedic sacrificial prescriptions and are in this sense a plausible starting point to introduce axioms for a logical system.

Axiom (A1) arises from three different principles; among them there is the following abstraction of a *nyāya* found in *Tantrarahasya* IV.4.3.3 (about which see *Freschi 2012*):

> If the accomplishment of X presupposes the accomplishment of Y, the obligation to perform X prescribes also Y.

[35] See *Chellas 1980* for a presentation of **S4** and other alethic modal systems.

Axiom (A2) arises from the so-called *principle of the half-hen*, which is implemented in different Mīmāṃsā contexts (e.g. Kumārila's subcommentary on *Commentary on Exegetic Aphorisms* 1.3.3); an abstract representation of it is:[36]

> Given that purposes Y and Z exclude each other, if one should use item X for the purpose Y, then it cannot be the case that one should use it at the same time for the purpose Z.

Finally, Axiom (A3) arises from a discussion (in ŚBh on *Exegetic Aphorisms* 6.1.25) on the eligibility to perform sacrifices, which can be abstracted as follows:

> If conditions X and Y are always equivalent, given the duty to perform Z under the condition X, the same duty applies under Y.

The notion of entailment involved in these principles is formalized in *Ciabattoni et al. 2015* in terms of *strict implication*: the claims 'p presupposes q', 'p and q exclude each other' and 'p and q are always equivalent' are respectively rendered as $\Box(p \rightarrow q)$, $\Box(p \rightarrow \neg q)$ and $\Box(p \equiv q)$.

Extracting properties of the operator $\mathcal{O}$

As already explained in Section 3.1, we decided to extract the axioms for $\mathcal{O}$ from the system **bMDL**. The extraction procedure is not trivial, since the axiomatic basis for **bMDL** provided in *Ciabattoni et al. 2015* includes no *specific principle* for $\mathcal{O}$ (namely, no principle in which $\mathcal{O}$ is the sole intensional operator). A general description of the procedure is the following: we first consider some relevant axioms and rules for dyadic $\mathcal{O}$ that are provable in **bMDL** (under a given translation) and show that they are sufficient to axiomatize the set of all theorems of **bMDL** in which the only intensional operator is $\mathcal{O}$, namely the *dyadic $\mathcal{O}$-fragment of **bMDL***. Then we also illustrate how one can obtain the *monadic $\mathcal{O}$-fragment of **bMDL***, by taking into account all theorems of **bMDL** in which $\mathcal{O}$ is the only intensional operator and always has the formula $\top$ as its precondition.

THEOREM A.1 *The schemes* $\neg(\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\neg\phi)$, $\mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\psi)$, *as well as the rules* $(\phi \equiv \psi)/(\mathcal{O}_\theta\phi \equiv \mathcal{O}_\theta\psi)$ *and* $(\theta \equiv \xi)/(\mathcal{O}_\theta\phi \equiv \mathcal{O}_\xi\phi)$, *hold in **bMDL**.*

Proof The conjunction of (A1) and (A2) entails $\vdash_{\mathbf{bMDL}} \Box(\phi \rightarrow \psi) \rightarrow (\mathcal{O}_\theta\phi \rightarrow (\mathcal{O}_\theta\psi \wedge \neg\mathcal{O}_\theta\neg\psi))$. By substitution, we get $\vdash_{\mathbf{bMDL}} \Box(\phi \rightarrow \phi) \rightarrow (\mathcal{O}_\theta\phi \rightarrow (\mathcal{O}_\theta\phi \wedge \neg\mathcal{O}_\theta\neg\phi))$; since $\vdash_{\mathbf{bMDL}} \Box(\phi \rightarrow \phi)$ ($\Box$ is a normal operator in this system), then by Modus Ponens $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta\phi \rightarrow (\mathcal{O}_\theta\phi \wedge \neg\mathcal{O}_\theta\neg\phi)$, whence $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta\phi \rightarrow \neg\mathcal{O}_\theta\neg\phi$, which is $\vdash_{\mathbf{bMDL}} \neg(\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\neg\phi)$. From $\vdash_{\mathbf{bMDL}} \Box(\phi \rightarrow \psi) \rightarrow (\mathcal{O}_\theta\phi \rightarrow (\mathcal{O}_\theta\psi \wedge \neg\mathcal{O}_\theta\neg\psi))$ we can also get, by substitution, $\vdash_{\mathbf{bMDL}} \Box((\phi \wedge \psi) \rightarrow \phi) \rightarrow (\mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta\phi \wedge \neg\mathcal{O}_\theta\neg\phi))$ and, since $\vdash_{\mathbf{bMDL}} \Box((\phi \wedge \psi) \rightarrow \phi)$, then $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta\phi \wedge \neg\mathcal{O}_\theta\neg\phi))$, which entails $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta(\phi \wedge \psi) \rightarrow \mathcal{O}_\theta\phi$. A parallel argument can be used to get $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta(\phi \wedge \psi) \rightarrow \mathcal{O}_\theta\psi$, hence $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\psi)$. Assuming $\vdash_{\mathbf{bMDL}} \phi \equiv \psi$, we get $\vdash_{\mathbf{bMDL}} \Box(\phi \equiv \psi)$, whence $\vdash_{\mathbf{bMDL}} \Box(\phi \rightarrow \psi) \wedge \Box(\psi \rightarrow \phi)$, from which one can infer, by (A1), $\vdash_{\mathbf{bMDL}} (\mathcal{O}_\theta\phi \rightarrow \mathcal{O}_\theta\psi) \wedge (\mathcal{O}_\theta\psi \rightarrow \mathcal{O}_\theta\phi)$, which is the same as $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta\phi \equiv \mathcal{O}_\theta\psi$. Finally, by (A3) one immediately gets $\vdash_{\mathbf{bMDL}} (\Box(\psi \equiv \theta) \wedge \mathcal{O}_\psi\phi) \rightarrow \mathcal{O}_\theta\phi$. Assuming $\vdash_{\mathbf{bMDL}} \theta \equiv \xi$ we obtain $\vdash_{\mathbf{bMDL}} \Box(\theta \equiv \xi)$, hence $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta\phi \rightarrow \mathcal{O}_\xi\phi$ and $\vdash_{\mathbf{bMDL}} \mathcal{O}_\xi\phi \rightarrow \mathcal{O}_\theta\phi$, whence $\vdash_{\mathbf{bMDL}} \mathcal{O}_\theta\phi \equiv \mathcal{O}_\xi\phi$. ■

We will hereafter call $\mathbf{EMD}_d$ (i.e. dyadic **EMD**)[37] the system obtained by extending a suitable axiomatic basis for the propositional calculus with the principles $\neg(\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\neg\phi)$, $\mathcal{O}_\theta(\phi \wedge \psi) \rightarrow (\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\psi)$, $(\phi \equiv \psi)/(\mathcal{O}_\theta\phi \equiv \mathcal{O}_\theta\psi)$ and $(\theta \equiv \xi)/(\mathcal{O}_\theta\phi \equiv \mathcal{O}_\xi\phi)$. Let $VAR = \{p, q, r, \ldots\}$ be a set of propositional variables, $\mathcal{L}(\mathbf{EMD}_d)$ denote the language of $\mathbf{EMD}_d$ over VAR and $\mathcal{L}(\mathbf{bMDL})$ the language of **bMDL** over VAR. We interpret $\mathbf{EMD}_d$ in neighborhood frames of kind $\mathfrak{F} = \langle W, \{N_\theta : \theta \in \mathcal{L}(\mathbf{EMD}_d)\}\rangle$ where W is a set of worlds and, for any $\theta \in \mathcal{L}(\mathbf{EMD}_d)$, $N_\theta : W \longrightarrow \wp(\wp(W))$ is a neighborhood function. A model over such frames is a structure $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$ s.t. V is a function mapping propositional variables to subsets of W. For any formula ϕ let $\|\phi\|^{\mathfrak{M}}$ be the truth-set of ϕ in $\mathfrak{M}$, namely $\{v \in W : \mathfrak{M}, v \vDash \phi\}$. Formulas are evaluated with reference to a world in a model and truth-conditions are as usual. In particular:

– $\mathfrak{M}, w \vDash \mathcal{O}_\theta\phi$ iff $\|\phi\|^{\mathfrak{M}} \in N_\theta(w)$.

Following *Chellas 1980*, we can claim that $\mathbf{EMD}_d$ is complete w.r.t. the class of neighborhood frames having the following properties:

– if $X \in N(w)$ and $X \subseteq Y \subseteq W$, then $Y \in N(w)$;

[36] It is noteworthy that according to our analysis of Maṇḍana's approach in Section 4, it would be more convenient to reformulate axiom (A2) as follows:

$$\Box(\theta \rightarrow \neg\xi) \rightarrow \neg(\mathcal{O}_\theta\phi \wedge \mathcal{O}_\xi\phi).$$

In this way, the antecedent would say something about triggering conditions rather than instruments (and the purposes mentioned in the *principle of the half hen* are, in accordance with Maṇḍana, triggering conditions). This move would have several consequences on the resulting system.

[37] The name **EMD** is taken from *Chellas 1980*.

- if $X \in N(w)$, then $\bar{X} \notin N(w)$;
- if $\vdash_{\mathbf{EMD}_d} \theta \equiv \xi$, then for all $X \in \wp(W)$, $X \in N_\theta(w)$ iff $X \in N_\xi(w)$.

The neighborhood frames defined in *Ciabattoni et al.* 2015 are structures of kind $\mathfrak{F} = \langle W, R, N \rangle$, where W is a set of worlds, $R \subseteq W \times W$ is an accessibility relation associated with $\Box$ and $N : W \longrightarrow \wp(\wp(W) \times \wp(W))$ a neighborhood function associated with $\mathcal{O}$. Models over such frames and truth-conditions for formulas are defined as usual, except for:

- $\mathfrak{M}, w \vDash \mathcal{O}_\theta \phi$ iff $(\|\phi\|^{\mathfrak{M}} \cap R(w), \|\theta\|^{\mathfrak{M}} \cap R(w)) \in N(w)$.

Notice that when R is a universal relation (i.e. $R = W \times W$), this condition can be simplified to

- $\mathfrak{M}, w \vDash \mathcal{O}_\theta \phi$ iff $(\|\phi\|^{\mathfrak{M}}, \|\theta\|^{\mathfrak{M}}) \in N(w)$.

bMDL is complete w.r.t. the class of neighborhood frames having the following properties:

- R is transitive and reflexive;
- if $(X, Y) \in N(w)$, then $X, Y \subseteq R(w)$, where $R(w) = \{v \in W : wRv\}$;
- if $(X, Z) \in N(w)$ and $X \subseteq Y \subseteq R(w)$, then $(Y, Z) \in N(w)$;
- if $(X, Z) \in N(w)$, then $(\bar{X} \cap R(w), Z) \notin N(w)$ (and this entails, given the previous properties, $(\emptyset, X) \notin N(w)$).

We now show that any model over a frame for $\mathbf{EMD}_d$ can be converted into a model over a frame for **bMDL**. First, we need to specify a translation function t mapping formulas of $\mathcal{L}(\mathbf{EMD}_d)$ to formulas of $\mathcal{L}(\mathbf{bMDL})$. Consider the following clauses:

- for any propositional variable $p \in VAR$, $t(p) = p$;
- $t(\neg\phi) = \neg t(\phi)$;
- $t(\phi \to \psi) = t(\phi) \to t(\psi)$;
- $t(\mathcal{O}_\theta \phi) = \mathcal{O}_{t(\theta)} t(\phi)$.

Let $\mathfrak{M} = \langle W, \{N_\theta : \theta \in \mathcal{L}(\mathbf{EMD}_d)\}, V \rangle$ be an arbitrary model over a frame for $\mathbf{EMD}_d$. We define a model $\mathfrak{M}^* = \langle W^*, R^*, N^*, V^* \rangle$ s.t. $W^* = W$, $R^* = W^* \times W^*$, $V^* = V$ and, for any world $w \in W^*$, $N^*(w)$ is the set of all pairs (X, Y) s.t.:

- $Y = \|t(\xi)\|^{\mathfrak{M}^*}$ for some $\xi \in \mathcal{L}(\mathbf{EMD}_d)$;
- $X \supseteq Z$ for some $Z \in \wp(W)$ s.t. $Z \in N_\xi(w)$.

We show now that the translation function t makes the evaluation of any formula $\phi \in \mathbf{EMD}_d$ at any world $w \in W$ invariant among $\mathfrak{M}$ and $\mathfrak{M}^*$.

THEOREM A.2 *For any world $w \in W$ and any formula $\phi \in \mathcal{L}(\mathbf{EMD}_d)$, we have $\mathfrak{M}, w \vDash \phi$ iff $\mathfrak{M}^*, w \vDash t(\phi)$.*

Proof We use an induction on the length of formulas in $\mathcal{L}(\mathbf{EMD}_d)$. Atomic and boolean cases are straightforward, given that $V^* = V$. The only relevant case is $\phi = \mathcal{O}_\theta \psi$ and, by induction hypothesis, we can assume that $\|t(\psi)\|^{\mathfrak{M}^*} = \|\psi\|^{\mathfrak{M}}$. Let $\mathfrak{M}, w \vDash \mathcal{O}_\theta \psi$; then $\|\psi\|^{\mathfrak{M}} \in N_\theta(w)$ and this entails, by definition of $N^*(w)$, that $(\|t(\psi)\|^{\mathfrak{M}^*}, \|t(\theta)\|^{\mathfrak{M}^*}) \in N^*(w)$, whence $\mathfrak{M}^*, w \vDash \mathcal{O}_{t(\theta)} t(\psi)$, i.e. $\mathfrak{M}^*, w \vDash t(\mathcal{O}_\theta \psi)$. Let $\mathfrak{M}, w \nvDash \mathcal{O}_\theta \psi$; then $\|\psi\|^{\mathfrak{M}} \notin N_\theta(w)$ and, since $N_\theta(w)$ is closed under supersets (by definition of models for $\mathbf{EMD}_d$), there is no $Z \in \wp(W)$ s.t. $Z \in N_\theta(w)$ and $Z \subseteq \|\psi\|^{\mathfrak{M}}$. From this one can infer $(\|t(\psi)\|^{\mathfrak{M}^*}, \|t(\theta)\|^{\mathfrak{M}^*}) \notin N^*(w)$ and $\mathfrak{M}^*, w \nvDash \mathcal{O}_{t(\theta)} t(\psi)$, i.e. $\mathfrak{M}^*, w \nvDash t(\mathcal{O}_\theta \psi)$. ■

The most important consequence of Theorem A.2 is that for any formula $\phi \in \mathcal{L}(\mathbf{EMD}_d)$, if ϕ is falsified at some world w in $\mathfrak{M}$, then $t(\phi)$ is falsified at the same world in $\mathfrak{M}^*$.

Finally, it remains to show that $\mathfrak{M}^*$ is a model (over a frame) for **bMDL**.

THEOREM A.3 *For any formula $\phi \in \mathcal{L}(\mathbf{bMDL})$, if $\vdash_{\mathbf{bMDL}} \phi$, then $\mathfrak{M}^* \vDash \phi$.*

We use an induction on the length of derivations showing that all axioms of **bMDL** are valid in $\mathfrak{M}^*$ and that rules of **bMDL** preserve validity in $\mathfrak{M}^*$. First, notice that in $\mathfrak{M}^*$ the accessibility relation R^* is universal, so all theorems of **S4** are valid. Consider axiom (A1), namely $(\Box(\phi \to \psi) \wedge \mathcal{O}_\theta \phi) \to \mathcal{O}_\theta \psi$. Assume that, for some $w \in W^*$, we have $\mathfrak{M}^*, w \vDash \Box(\phi \to \psi) \wedge \mathcal{O}_\theta \phi$. This means that $R^*(w) \subseteq \|\phi \to \psi\|^{\mathfrak{M}^*}$, whence $\|\phi\|^{\mathfrak{M}^*} \cap R^*(w) \subseteq \|\psi\|^{\mathfrak{M}^*} \cap R^*(w)$. However, given that for any formula χ, $\|\chi\|^{\mathfrak{M}^*} \cap R^*(w) = \|\chi\|^{\mathfrak{M}^*}$, then $\|\phi\|^{\mathfrak{M}^*} \subseteq \|\psi\|^{\mathfrak{M}^*}$. We know that $\mathfrak{M}^*, w \vDash \mathcal{O}_\theta \phi$, so $(\|\phi\|^{\mathfrak{M}^*}, \|\theta\|^{\mathfrak{M}^*}) \in N^*(w)$ and this means (by definition of N^*) that there is some $\xi \in \mathcal{L}(\mathbf{EMD}_d)$ and some $Z \in \wp(W)$ s.t. $Z \in N_\xi(w)$, $Z \subseteq \|\phi\|^{\mathfrak{M}^*}$ and $t(\xi) = \|\theta\|^{\mathfrak{M}^*}$. From this one can infer $Z \subseteq \|\psi\|^{\mathfrak{M}^*}$, which is enough to get $\mathfrak{M}^*, w \vDash \mathcal{O}_\theta \psi$.

Consider axiom (A2), namely $\Box(\psi \to \neg\phi) \to \neg(\mathcal{O}_\theta\phi \wedge \mathcal{O}_\theta\psi)$. Assume that, for some $w \in W^*$, we have $\mathfrak{M}^*, w \vDash \Box(\psi \to \neg\phi) \wedge \mathcal{O}_\theta\psi$; then, we get $R^*(w) \subseteq \|\psi \to \neg\phi\|^{\mathfrak{M}^*}$, whence $\|\psi\|^{\mathfrak{M}^*} \subseteq \overline{\|\phi\|^{\mathfrak{M}^*}}$. Furthermore, there is some $\xi \in \mathcal{L}(\mathbf{EMD}_d)$ and some $Z \in \wp(W)$ s.t. $Z \in N_\xi(w)$, $Z \subseteq \|\psi\|^{\mathfrak{M}^*}$ and $t(\xi) = \|\theta\|^{\mathfrak{M}^*}$. If it were the case that $\mathfrak{M}^*, w \vDash \mathcal{O}_\theta\phi$, then we would have, for some $Q \in \wp(W)$, $Q \in N_\xi(w)$ and $Q \subseteq \|\phi\|^{\mathfrak{M}^*}$. However, notice that $Z \subseteq \overline{\|\phi\|^{\mathfrak{M}^*}}$; moreover, we know that $Z \in N_\xi(w)$. The definition of models for $\mathbf{EMD}_d$ tells us that $N_\xi(w)$ is closed under supersets; thus (given that $W = W^*$ and so $\|\phi\|^{\mathfrak{M}^*}, \overline{\|\phi\|^{\mathfrak{M}^*}} \subseteq W$), if it were the case that $Q \in N_\xi(w)$, one would get both $\|\phi\|^{\mathfrak{M}^*} \in N_\xi(w)$ and $\overline{\|\phi\|^{\mathfrak{M}^*}} \in N_\xi(w)$, which is impossible, since (by definition of models for $\mathbf{EMD}_d$), $N_\xi(w)$ cannot contain complementary sets.

Consider axiom (A3), namely $(\Box((\psi \to \theta) \wedge (\theta \to \psi)) \wedge (\mathcal{O}_\psi\phi)) \to \mathcal{O}_\theta\phi$, which can be simplified to $\Box(\psi \equiv \theta) \to (\mathcal{O}_\psi\phi \equiv \mathcal{O}_\theta\phi)$. Suppose $\mathfrak{M}^*, w \vDash \Box(\psi \equiv \theta)$ for some $w \in W^*$. Then, we get $\mathfrak{M}^*, w \vDash \mathcal{O}_\psi\phi$ iff $(\|\phi\|^{\mathfrak{M}^*}, \|\psi\|^{\mathfrak{M}^*}) \in N^*(w)$ iff (given that $\|\psi\|^{\mathfrak{M}^*} = \|\theta\|^{\mathfrak{M}^*}$ by assumption) $(\|\phi\|^{\mathfrak{M}^*}, \|\theta\|^{\mathfrak{M}^*}) \in N^*(w)$ iff $\mathfrak{M}^*, w \vDash \mathcal{O}_\theta\phi$; thus, $\mathfrak{M}^*, w \vDash \mathcal{O}_\psi\phi \equiv \mathcal{O}_\theta\phi$, which is enough. The fact that necessitation and Modus Ponens (the only rules of **bMDL**) preserve validity in $\mathfrak{M}^*$ is straightforward. ■

Taken together, Theorems A.2 and A.3 entail that for any formula $\phi \in \mathcal{L}(\mathbf{EMD}_d)$, if ϕ is falsifiable in some model for $\mathbf{EMD}_d$, then there is a model for **bMDL** falsifying $t(\phi)$. Given the characterization results for $\mathbf{EMD}_d$ and **bMDL**, this means that for any $\phi \in \mathcal{L}(\mathbf{EMD}_d)$, if $\nvdash_{\mathbf{EMD}_d} \phi$, then $\nvdash_{\mathbf{bMDL}} t(\phi)$. Since, by Theorem A.1, we also have that if $\vdash_{\mathbf{EMD}_d} \phi$, then $\vdash_{\mathbf{bMDL}} t(\phi)$, then we can claim that $\mathbf{EMD}_d$ is the *dyadic* $\mathcal{O}$-fragment of **bMDL** under the translation function t.

As long as the *monadic* $\mathcal{O}$-fragment of **bMDL** is concerned, let **EMD** be the monadic version of $\mathbf{EMD}_d$; more precisely, **EMD** is the system obtained by extending a suitable axiomatic basis for the propositional calculus with the principles $\neg(\mathcal{O}\phi \wedge \mathcal{O}\neg\phi)$, $\mathcal{O}(\phi \wedge \psi) \to (\mathcal{O}\phi \wedge \mathcal{O}\psi)$ and $(\phi \equiv \psi)/(\mathcal{O}\phi \equiv \mathcal{O}\psi)$. Now, we can define a translation function f mapping formulas of $\mathcal{L}(\mathbf{EMD})$, the language of **EMD**, to formulas of $\mathcal{L}(\mathbf{EMD}_d)$, the language of $\mathbf{EMD}_d$ (assuming, as usual, that the two languages are based on the same set of propositional variables VAR). Consider the following clauses (where $\top$ stands for an arbitrary tautology):

- for any propositional variable $p \in VAR$, $f(p) = p$;
- $f(\neg\phi) = \neg f(\phi)$;
- $f(\phi \to \psi) = f(\phi) \to t(\psi)$;
- $f(\mathcal{O}\phi) = \mathcal{O}_\top f(\phi)$.

The intuition behind the last clause is that something is unconditionally obligatory iff it is obligatory under any condition which is trivially the case ($\top$): saying that one (unconditionally) ought to pay taxes is the same as saying that one ought to pay taxes given that it rains or it does not rain.[38] Clearly, under the translation function f **EMD** corresponds to what we can call the $\mathcal{O}_\top$-fragment of $\mathbf{EMD}_d$, namely the system obtained by extending a suitable axiomatic basis of the propositional calculus with the principles $\neg(\mathcal{O}_\top\phi \wedge \mathcal{O}_\top\neg\phi)$, $\mathcal{O}_\top(\phi \wedge \psi) \to (\mathcal{O}_\top\phi \wedge \mathcal{O}_\top\psi)$, $(\phi \equiv \psi)/(\mathcal{O}_\top\phi \equiv \mathcal{O}_\top\psi)$ and $(\top \equiv \top)/(\mathcal{O}_\top\phi \equiv \mathcal{O}_\top\phi)$. Indeed, using the translation function f one can easily show that the first three principles are just the images under f of the principles $\neg(\mathcal{O}\phi \wedge \mathcal{O}\neg\phi)$, $\mathcal{O}(\phi \wedge \psi) \to (\mathcal{O}\phi \wedge \mathcal{O}\psi)$ and $(\phi \equiv \psi)/(\mathcal{O}\phi \equiv \mathcal{O}\psi)$; furthermore, the fourth principle, namely $(\top \equiv \top)/(\mathcal{O}_\top\phi \equiv \mathcal{O}_\top\phi)$, is equivalent to the image under f of $\mathcal{O}\phi \equiv \mathcal{O}\phi$, which is a tautology. Theorems A.1–A.3 and the translation function t used above in this section guarantee that the $\mathcal{O}_\top$-fragment of $\mathbf{EMD}_d$ is also the $\mathcal{O}_\top$-fragment of **bMDL**. Finally consider the translation function g mapping formulas of **EMD** to formulas of **bMDL** and resulting from the combination of f and t: for any $\phi \in \mathcal{L}(\mathbf{EMD})$, $g(\phi) = t(f(\phi))$ and $t(f(\phi)) \in \mathcal{L}(\mathbf{bMDL})$ (by definition of t and f). We immediately get that g satisfies the following clauses:

- for any propositional variable $p \in VAR$, $g(p) = p$;
- $g(\neg\phi) = \neg g(\phi)$;
- $g(\phi \to \psi) = g(\phi) \to g(\psi)$;
- $g(\mathcal{O}\phi) = \mathcal{O}_\top g(\phi)$.

From this one can conclude that **EMD** is the *monadic* $\mathcal{O}$-fragment of **bMDL** under the translation function g.

On the interpretation of the operator $\mathcal{O}$

The system **bMDL**, from which we extracted the principles for $\mathcal{O}$, has among its theorems all instances of the schema $\mathcal{O}_\theta\phi \to \mathcal{O}_\theta(\phi \vee \psi)$. This schema is often taken to be puzzling for deontic reasoning, since it allows one to derive *Ross's paradox* (see, e.g. *Hilpinen and McNamara* 2013): if p stands for the proposition expressed by the sentence 'Mark posts the letter' and q for the proposition expressed by the sentence 'Mark burns the letter', then one can read the formula $\mathcal{O}_\top(p \vee q)$ as saying that Mark ought to post the letter or burn it. The problem is that in **bMDL**, as well as in its extensions treated in the present article, one can infer $\mathcal{O}_\top(p \vee q)$ from $\mathcal{O}_\top p$, namely from the fact that Mark ought to post the letter (see also the monadic translation of the two formulas at issue under the function g described above). At a first glance, this seems to be a relevant drawback of logical

[38] The function f represents a standard translation of languages with monaidc deontic operators into languages with dyadic deontic operators. See, for instance, the Appendix to Section 8.5 in *Hilpinen and McNamara* 2013.

systems used to represent deontic modalities; however, a closer look at the intended semantics of $\mathcal{O}$ reveals that the schema $\mathcal{O}_\theta \phi \rightarrow \mathcal{O}_\theta(\phi \vee \psi)$ can be interpreted in a plausible way.

Indeed, in this article the operator $\mathcal{O}$ has to be read as 'in every situation in which all strong Vedic duties are observed ...'; such reading applies both to the monadic and to the dyadic case, as illustrated in Section 3. Now, the truth of $\mathcal{O}_\theta \phi$ in a situation w means that ϕ describes a state-of-affairs holding in every situation where all strong Vedic duties applying to w are observed and where the condition expressed by θ is fulfilled. Moreover, it is clearly appropriate to say that the state-of-affairs described by $\phi \vee \psi$ generalizes the state-of-affairs described by ϕ; therefore, ϕ describes something which is sufficient to have $\phi \vee \psi$ true. From this one can conclude that $\phi \vee \psi$ cannot be false in any situation in which all strong Vedic duties applying to w are observed and the condition θ is fulfilled. But this is exactly the same as saying that $\mathcal{O}_\theta(\phi \vee \psi)$ is true at w.[39]

[39] For a broader discussion and criticism of Ross's paradox, the reader is referred to *Castañeda 1981* and *Åqvist 1987*. Our argument is in line with the criticism provided in *Cocchiarella 2015*, which aims at defending the use of normal (hence, monotonic) systems of deontic logic.